

## Representing the Graviton Self-Energy on de Sitter Background

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### ABSTRACT

We derive a noncovariant but simple representation for the self-energy of a conformally transformed graviton field on the cosmological patch of de Sitter. Our representation involves four structure functions, as opposed to the two that would be necessary for a manifestly de Sitter invariant representation. We work out what the four structure functions are for the one loop correction due to a massless, minimally coupled scalar. And we employ the result to work out what happens to dynamical gravitons.

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# 1 Introduction

Inflation produces vast ensembles of infrared scalars and gravitons which are thought to be the source of primordial perturbations [1]. The primary perturbations are a tree order effect, which means that how they interact among themselves and with other particles is a loop correction. One studies these loop effects by first computing the appropriate 1PI (one-particle-irreducible) 2-point function and then using it to quantum-correct the linearized effective field equation for the particle in question [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

If the particle we are studying has nonzero spin then its 1PI 2-point function must carry tensor or spinor indices. For example, the self-energy of a Dirac fermion has 16 bi-spinor components, the vacuum polarization possesses 16 bi-vector components, while the graviton self-energy contains 100 bi-tensor components. Although it would not be wrong to report results for each component separately, experience with quantum field theory on flat space shows that this is wasteful and that it obscures important features of the dynamics. For example, the combination of gauge invariance and Poincaré invariance implies that the vacuum polarization of flat space can be expressed in terms of a single scalar structure function,

$$i \left[ {}^\mu \Pi_{\text{flat}}^\nu \right] (x; x') = \left[ \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^2 \right] i \Pi \left( (x - x')^2 \right), \quad (1)$$

where  $\eta^{\mu\nu}$  is the Minkowski metric and  $(x - x')^2 \equiv \eta_{\mu\nu} (x - x')^\mu (x - x')^\nu$ .

How much the 1PI 2-point function can be simplified depends partly upon linearized gauge invariances and partly on isometries. The background geometry appropriate for inflationary cosmology is homogeneous, isotropic and spatially flat,

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} = a^2 \left[ -d\eta^2 + d\vec{x} \cdot d\vec{x} \right]. \quad (2)$$

For many purposes it is also desirable to take the de Sitter limit in which the scale factor becomes  $a = -1/H\eta$ , with constant  $H$ . That would introduce four additional isometries, however, one has to bear in mind that de Sitter can only be an approximation when studying primordial inflation. There can also be important de Sitter breaking effects from inflationary scalars [13, 2, 3, 5] and gravitons [14, 15, 16, 17, 18, 6, 11, 12]. And even when exact de Sitter invariance is present the cost of making it manifest can be prohibitive [19, 9].

Our goal is to develop a simple form for representing the graviton self-energy on de Sitter background. The contributions to this from many sorts of matter fields are de Sitter invariant, and a fully de Sitter invariant representation using two structure functions was derived in a previous study of the one loop graviton self-energy from a massless, minimally coupled scalar [9]. However, this representation turned out to be horrifically complicated [9], and tedious to employ [10].

We also suspect the de Sitter invariant representation for gravitons might give misleading results. It is disturbingly similar to what we recently found for the de Sitter invariant contribution to the vacuum polarization from a massive scalar [19]. The original computation of this effect was reported using a noncovariant representation in which there are two structure functions and only the isometries of homogeneity and isotropy are manifest [4]. With that representation the dynamics are transparent and it was simple to show that dynamical photons become massive. With our de Sitter invariant representation the dynamics are cumbersome and the mass contribution to the effective field equation takes the form of an integral over the initial value surface. These surface terms are usually irrelevant but we were (at length) able to recognize this one as a local Proca mass term using Green's second identity [19]. Had that surface term been discarded we would have erroneously reached the null conclusion which *was* reached for the graviton using the same, cumbersome and confusing de Sitter invariant formalism and dropping the same sort of surface term [10].

In section 2 we show that linearized gauge invariance, with only the isometries of homogeneity and isotropy, results in four structure functions for the graviton self-energy, rather than the two of a fully de Sitter invariant representation. We explicitly construct the 4-function representation in section 3. In section 4 we work out the four structure functions for the one loop contribution from a massless, minimally coupled scalar. The new representation is employed in section 5 to re-examine the question of quantum corrections to dynamical gravitons. Our conclusions comprise section 6.

## 2 Counting and Conformal Rescaling

The point of this section is to count the number and type of structure functions which are required to represent the graviton self-energy when we relax the assumption of full de Sitter invariance to just the cosmological isometries

of homogeneity and isotropy. We begin by describing the coordinate system of the cosmological patch and the natural basis vectors on it. The number of structure functions is just the number of homogeneous and isotropic basis tensors minus the number of constraints implied by transversality. An important subtlety is that the simple representation we aim to derive is for the conformally rescaled graviton self-energy. The section closes by laying out precisely what transversality implies for this quantity.

## 2.1 Coordinates and basis vectors

We work on open conformal coordinates (in  $D$  spacetime dimensions to facilitate dimensional regularization),

$$ds^2 = a^2(\eta) \left[ -d\eta^2 + d\vec{x} \cdot d\vec{x} \right] \quad , \quad a(\eta) = -\frac{1}{H\eta} \quad , \quad (3)$$

where the coordinate ranges are,

$$-\infty < x^0 \equiv \eta < 0 \quad , \quad -\infty < x^i < +\infty \quad , \quad i = 1, 2, \dots, D-1 \quad . \quad (4)$$

When a bi-tensor density such as the graviton self-energy is de Sitter invariant it can be expressed using the invariant length  $\ell(x; x')$ . For quantum field theory computations it is most convenient to employ the de Sitter length function  $y(x; x') \equiv 4 \sin^2[\frac{1}{2} H \ell(x; x')]$ ,

$$y(x; x') \equiv H^2 a a' \left[ \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\varepsilon)^2 \right] \quad , \quad (5)$$

where  $a \equiv a(\eta)$  and  $a' \equiv a(\eta')$ . When only homogeneity and isotropy are present one must allow additional dependence upon two combinations of the scale factors,

$$u(x; x') \equiv \ln(aa') \quad , \quad v(x; x') \equiv \ln\left(\frac{a}{a'}\right) \quad . \quad (6)$$

A convenient basis of de Sitter invariant bi-tensors can be formed using products of the metrics at  $x^\mu$  and  $x'^\mu$ , along with the first three derivatives of  $y(x; x')$ ,

$$\partial_\mu y = aH \left( \delta_\mu^0 y + 2a' H \Delta x_\mu \right) \quad , \quad \partial'_\nu y = a' H \left( \delta_\nu^0 y - 2a H \Delta x_\nu \right) \quad , \quad (7)$$

$$\partial_\mu \partial'_\nu y = a a' H^2 \left( \delta_\mu^0 \delta_\nu^0 y - 2a \delta_\mu^0 H \Delta x_\nu + 2a' \delta_\nu^0 H \Delta x_\mu - 2\eta_{\mu\nu} \right) \quad , \quad (8)$$

where  $\Delta x_\mu \equiv \eta_{\mu\nu}(x - x')^\nu$ . It is straightforward to show that covariant derivatives and/or contractions of these basis tensors produce only tensors within the basis [20]. When only homogeneity and isotropy are present one must include the first derivatives of  $u(x; x')$  [16],

$$\partial_\mu u = aH\delta_\mu^0 \quad , \quad \partial'_\nu u = a'H\delta_\nu^0 . \quad (9)$$

Derivatives of  $v(x; x')$  are unnecessary because  $\partial_\mu v = +\partial_\mu u$  and  $\partial'_\nu v = -\partial'_\nu u$ . Acting covariant derivatives on any element of (7-9), or contracting any two elements, produces sums of products of more basis elements [16].

## 2.2 Counting the structure functions

If the full metric is  $g_{\mu\nu}^{\text{full}}$  and the metric of the de Sitter background is  $g_{\mu\nu} = a^2\eta_{\mu\nu}$  then the graviton field of the invariant representation is,

$$\chi_{\mu\nu}(x) \equiv \frac{g_{\mu\nu}^{\text{full}}(x) - g_{\mu\nu}(x)}{\kappa} \quad , \quad \kappa^2 \equiv 16\pi G . \quad (10)$$

The self-energy of this field is a transverse bi-tensor density,

$$D_\mu \left[ {}^{\mu\nu}\Sigma_\chi^{\rho\sigma} \right](x; x') = 0 = D'_\rho \left[ {}^{\mu\nu}\Sigma_\chi^{\rho\sigma} \right](x; x') , \quad (11)$$

where  $D_\mu$  and  $D'_\rho$  stand for the covariant derivatives with respect to  $x^\mu$  and  $x'^\rho$  computed using the affine connection of the de Sitter background,

$$\Gamma^\rho_{\mu\nu}(x) = aH \left( \delta^\rho_\mu \delta^0_\nu + \delta^\rho_\nu \delta^0_\mu - \eta^{\rho 0} \eta_{\mu\nu} \right) = \delta^\rho_\mu u_{,\nu} + \delta^\rho_\nu u_{,\mu} - u^{,\rho} g_{\mu\nu} . \quad (12)$$

The self-energy is also invariant under interchange of coordinates and index groups,

$$-i \left[ {}^{\mu\nu}\Sigma_\chi^{\rho\sigma} \right](x; x') = -i \left[ {}^{\rho\sigma}\Sigma_\chi^{\mu\nu} \right](x'; x) . \quad (13)$$

If the graviton self-energy is manifestly de Sitter invariant it must consist of a linear combination of five tensors,

$$\begin{aligned} & D^\mu D'^{(\rho} y D'^{\sigma)} D^\nu y \quad , \quad D^{(\mu} y D^{\nu)} D'^{(\rho} y D'^{\sigma)} y \quad , \quad D^\mu y D^\nu y D'^{\rho} y D'^{\sigma} y \quad , \\ & H^2 \left( g^{\mu\nu} D'^{\rho} y D'^{\sigma} y + D^\mu y D^\nu y g'^{\rho\sigma} \right) \quad , \quad H^4 g^{\mu\nu} g'^{\rho\sigma} . \end{aligned} \quad (14)$$

Here and henceforth indices which are enclosed in parentheses are symmetrized. Transversality (11) means the covariant divergence  $D_\mu$  vanishes, which implies relations proportional to the three tensors,

$$D^\nu D'^{(\rho} y D'^{\sigma)} y \ , \ D^\nu y D'^\rho y D'^\sigma y \ , \ D^\nu y g'^{\rho\sigma} \ . \quad (15)$$

Hence we require  $5 - 3 = 2$  structure functions to make de Sitter invariance manifest when it is present. One of these is associated with a transverse-traceless tensor structure that mixes the  $x^\mu$  and  $x'^\mu$  index groups while the other tensor structure is diagonal [9].

If the only isometries are homogeneity and isotropy the most general reflection invariant bi-tensor requires nine more tensors in addition to those of (14) [17],

$$\begin{aligned} & \left( D^{(\mu} y D^{\nu)} D'^{(\rho} y D'^{\sigma)} u + D^{(\mu} u D^{\nu)} D'^{(\rho} y D'^{\sigma)} y \right) \ , \ D^{(\mu} u D^{\nu)} D'^{(\rho} y D'^{\sigma)} u \ , \\ & \left( D^\mu y D^\nu y D'^\rho u D'^\sigma u + D^\mu u D^\nu u D'^\rho y D'^\sigma y \right) \ , \ D^{(\mu} y D^{\nu)} u D'^{(\rho} y D'^{\sigma)} u \ , \\ & \left( D^{(\mu} y D^{\nu)} u D'^\rho u D'^\sigma u + D^\mu u D^\nu u D'^{(\rho} u D'^{\sigma)} y \right) \ , \ D^\mu u D^\nu u D'^\rho u D'^\sigma u \ , \\ & H^2 \left( D^{(\mu} y D^{\nu)} u g'^{\rho\sigma} + g^{\mu\nu} D'^{(\rho} y D'^{\sigma)} u \right) \ , \ H^2 \left( D^\mu u D^\nu u g'^{\rho\sigma} + g^{\mu\nu} D'^\rho u D'^\sigma u \right) \ , \\ & H^2 \left( D^\mu u D^\nu u g'^{\rho\sigma} - g^{\mu\nu} D'^\rho u D'^\sigma u \right) \ . \end{aligned} \quad (16)$$

Transversality (11) implies relations proportional to seven tensors in addition to those of (15),

$$\begin{aligned} & D^\nu D'^{(\rho} y D'^{\sigma)} u \ , \ D^\nu y D'^{(\rho} y D'^{\sigma)} u \ , \ D^\nu y D'^\rho u D'^\sigma u \ , \\ & D^\nu u D'^\rho y D'^\sigma y \ , \ D^\nu u D'^{(\rho} y D'^{\sigma)} u \ , \ D^\nu u D'^\rho u D'^\sigma u \ , \ D^\nu u g'^{\rho\sigma} \ . \end{aligned} \quad (17)$$

Hence there must be  $14 - 10 = 4$  structure functions when only homogeneity and isotropy are manifest. We might guess that two of them will be associated with transverse-traceless tensor structures which mix index groups while the remaining two are diagonal.

### 2.3 Conformal rescaling

Although mathematical physicists prefer to consider the “graviton field” to be the quantity  $\chi_{\mu\nu}$  defined in expression (10), it has long been known that

the simplest Feynman rules arise for the conformally rescaled graviton field [14],

$$h_{\mu\nu}(x) \equiv \frac{g_{\mu\nu}^{\text{full}}(x) - g_{\mu\nu}(x)}{\kappa a^2} = a^{-2} \times \chi_{\mu\nu}(x) . \quad (18)$$

This is the variable for which all of the existing fully dimensionally regulated graviton loop computations have been made [6, 7, 8, 11, 12]. Expression (18) fixes the relation between the propagator of  $\chi_{\mu\nu}$  and that of  $h_{\mu\nu}$ ,

$$i \left[ {}_{\mu\nu} \Delta_{\rho\sigma}^{\chi} \right] (x; x') \equiv \left\langle \Omega_0 \left| T \left[ \chi_{\mu\nu}(x) \chi_{\rho\sigma}(x') \right] \right| \Omega_0 \right\rangle , \quad (19)$$

$$= (aa')^2 \times \left\langle \Omega_0 \left| T \left[ h_{\mu\nu}(x) h_{\rho\sigma}(x') \right] \right| \Omega_0 \right\rangle \equiv (aa')^2 \times i \left[ {}_{\mu\nu} \Delta_{\rho\sigma} \right] (x; x') . \quad (20)$$

To infer the corresponding relation between the self-energies of  $\chi_{\mu\nu}$  and  $h_{\mu\nu}$ , it suffices to compare the one loop corrections to the full propagators. For the field  $\chi_{\mu\nu}$  we have,

$$\begin{aligned} i \left[ {}_{\mu\nu} \Delta_{\rho\sigma}^{\chi 1} \right] (x; x') \\ = \int d^D z \int d^D z' i \left[ {}_{\mu\nu} \Delta_{\alpha\beta}^{\chi} \right] (x; z) \times -i \left[ {}^{\alpha\beta} \Sigma_{\chi}^{\kappa\lambda} \right] (z; z') \times i \left[ {}_{\kappa\lambda} \Delta_{\rho\sigma}^{\chi} \right] (z'; x') . \end{aligned} \quad (21)$$

The corresponding expression for the one loop correction to the  $h_{\mu\nu}$  propagator is,

$$\begin{aligned} i \left[ {}_{\mu\nu} \Delta_{\rho\sigma}^1 \right] (x; x') \\ = \int d^D z \int d^D z' i \left[ {}_{\mu\nu} \Delta_{\alpha\beta} \right] (x; z) \times -i \left[ {}^{\alpha\beta} \Sigma^{\kappa\lambda} \right] (z; z') \times i \left[ {}_{\kappa\lambda} \Delta_{\rho\sigma} \right] (z'; x') . \end{aligned} \quad (22)$$

Because expression (21) must be  $(aa')^2$  times expression (22) the two self-energies must be related as,

$$-i \left[ {}^{\mu\nu} \Sigma_{\chi}^{\rho\sigma} \right] (x; x') = (aa')^{-2} \times -i \left[ {}^{\mu\nu} \Sigma^{\rho\sigma} \right] (x; x') . \quad (23)$$

## 2.4 Consequences of transversality

The self-energy of  $\chi_{\mu\nu}$  is a bi-tensor density. This means it can be expressed as proper bi-tensor times the measure factors from  $x^\mu$  and  $x'^\mu$ ,

$$-i \left[ {}^{\mu\nu} \Sigma_{\chi}^{\rho\sigma} \right] (x; x') = \sqrt{-g(x)} \sqrt{-g(x')} \times \left[ {}^{\mu\nu} T^{\rho\sigma} \right] (x; x') . \quad (24)$$

Transversality (11) means that the covariant divergence of  $[\mu\nu T^{\rho\sigma}](x; x')$  vanishes,

$$D_\mu [\mu\nu T^{\rho\sigma}] = \partial_\mu [\mu\nu T^{\rho\sigma}] + \Gamma^\alpha_{\alpha\mu} [\mu\nu T^{\rho\sigma}] + \Gamma^\nu_{\mu\alpha} [\mu\alpha T^{\rho\sigma}] = 0 . \quad (25)$$

Because  $\partial_\mu \sqrt{-g} = \sqrt{-g} \Gamma^\alpha_{\alpha\mu}$  we can re-express transversality (11) using the ordinary derivative,

$$\partial_\mu [\mu\nu \Sigma^\rho_\chi](x; x') + \Gamma^\nu_{\alpha\beta}(x) [\alpha\beta \Sigma^\rho_\chi](x; x') = 0 . \quad (26)$$

Now make use of relations (12) and (23) to conclude that the self-energy of the field  $h_{\mu\nu}$  obeys,

$$\partial_\mu [\mu\nu \Sigma^{\rho\sigma}](x; x') - D^\nu u g_{\alpha\beta}(x) \times [\alpha\beta \Sigma^{\rho\sigma}](x; x') = 0 . \quad (27)$$

### 3 The Four Projectors

The aim of this section is to derive an explicit expression for the self-energy of  $h_{\mu\nu}$  in terms of the four structure functions we have just seen are needed when only homogeneity and isotropy are present. We are guided by two facts, the first of which is the form taken by the flat space limit,

$$-i [\mu\nu \Sigma^{\rho\sigma}_{\text{flat}}] = \Pi^{\mu\nu} \Pi^{\rho\sigma} f_0((x-x')^2) + \left[ \Pi^{\mu(\rho} \Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu} \Pi^{\rho\sigma}}{D-1} \right] f_2((x-x')^2) , \quad (28)$$

where  $\Pi^{\mu\nu} \equiv \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^2$ . The structure functions  $f_0$  and  $f_2$  are usually labeled *spin zero* and *spin two*, respectively. The second fact is the simple noncovariant form of the vacuum polarization on de Sitter [3, 4, 19],

$$i [\mu \Pi^\nu] = \left( \eta^{\mu\nu} \eta^{\rho\sigma} - \eta^{\mu\sigma} \eta^{\nu\rho} \right) \partial_\rho \partial'_\sigma F(x; x') + \left( \overline{\eta}^{\mu\nu} \overline{\eta}^{\rho\sigma} - \overline{\eta}^{\mu\sigma} \overline{\eta}^{\nu\rho} \right) \partial_\rho \partial'_\sigma G(x; x') , \quad (29)$$

where an overlined tensor indicates suppression of its temporal components,  $\overline{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta^0_\mu \delta^0_\nu$ . The structure function  $F(x; x')$  is the one which survives in the flat space limit;  $G(x; x')$  is less divergent and vanishes with  $H$ .

These two hints motivate a representation of the form,

$$\begin{aligned} -i [\mu\nu \Sigma^{\rho\sigma}](x; x') &= \mathcal{F}^{\mu\nu}(x) \times \mathcal{F}^{\rho\sigma}(x') [F_0(x; x')] \\ &+ \mathcal{G}^{\mu\nu}(x) \times \mathcal{G}^{\rho\sigma}(x') [G_0(x; x')] + \mathcal{F}^{\mu\nu\rho\sigma} [F_2(x; x')] + \mathcal{G}^{\mu\nu\rho\sigma} [G_2(x; x')] . \end{aligned} \quad (30)$$



The idea is that the two “F” terms should represent appropriate de Sitter generalizations of the flat space result (28) while the two “G” terms should be essentially “spatial” like the  $G$  term in the vacuum polarization (29). Each of the four terms should separately respect transversality (27). So the second order differential operators  $\mathcal{F}^{\mu\nu}$  and  $\mathcal{G}^{\mu\nu}$  must obey,

$$\partial_\mu \mathcal{F}^{\mu\nu} + aH\delta_0^\nu \eta_{\alpha\beta} \mathcal{F}^{\alpha\beta} = 0 = \partial_\mu \mathcal{G}^{\mu\nu} + aH\delta_0^\nu \eta_{\alpha\beta} \mathcal{G}^{\alpha\beta} . \quad (31)$$

The differential operators  $\mathcal{F}^{\mu\nu\rho\sigma}$  and  $\mathcal{G}^{\mu\nu\rho\sigma}$  should each contain two primed and two unprimed derivatives and should each be separately transverse and traceless,

$$\partial_\mu \mathcal{F}^{\mu\nu\rho\sigma} = 0 = \partial_\mu \mathcal{G}^{\mu\nu\rho\sigma} , \quad \eta_{\mu\nu} \mathcal{F}^{\mu\nu\rho\sigma} = 0 = \eta_{\mu\nu} \mathcal{G}^{\mu\nu\rho\sigma} . \quad (32)$$

### 3.1 The scalar projectors $\mathcal{F}^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$

We construct the two scalar projectors by making an initial ansatz and then using the transversality relation (31) to determine the free parameters. Because we want the flat space limit of  $\mathcal{F}^{\mu\nu}$  to be  $\Pi^{\mu\nu} = \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^2$ , our ansatz for it is,

$$\mathcal{F}^{\mu\nu} = \partial^\mu \partial^\nu + 2f_1 aH \delta_0^{(\mu} \partial^{\nu)} + f_2 a^2 H^2 \delta_0^\mu \delta_0^\nu - \eta^{\mu\nu} \left[ \partial^2 + f_3 aH \partial_0 + f_4 a^2 H^2 \right] . \quad (33)$$

Equation (31) determines  $f_1 = f_3 = f_4 = (D-1)$  and  $f_2 = (D-2)(D-1)$ . Because the structure functions tend to carry a factor of  $a^{D-2}$  it is useful to note,

$$\mathcal{F}^{\mu\nu} = a^{D-2} \left[ \partial^\mu \partial^\nu + 2aH \delta_0^{(\mu} \partial^{\nu)} - \eta^{\mu\nu} \left[ \partial^2 - (D-3)aH \partial_0 + (D-1)a^2 H^2 \right] \right] a^{-(D-2)} . \quad (34)$$

In this form we can express the trace in terms of the invariant scalar d’Alembertian  $\square = a^{-D} \partial_\mu (a^{D-2} \eta^{\mu\nu} \partial_\nu)$ ,

$$a^D \times \mathcal{F} \times a^{-(D-2)} \equiv \eta_{\mu\nu} \mathcal{F}^{\mu\nu} = -(D-1)a^D \times \left[ \square + DH^2 \right] \times a^{-(D-2)} . \quad (35)$$

Because we want  $\mathcal{G}^{\mu\nu}$  to be “essentially spatial” our ansatz for it is,

$$\mathcal{G}^{\mu\nu} = \bar{\partial}^\mu \bar{\partial}^\nu + 2g_1 aH \delta_0^{(\mu} \bar{\partial}^{\nu)} + g_2 a^2 H^2 \delta_0^\mu \delta_0^\nu - \bar{\eta}^{\mu\nu} \left[ \nabla^2 + g_3 aH \partial_0 + g_4 a^2 H^2 \right] , \quad (36)$$

where we remind the reader that a line over a vector indicates suppression of its temporal components,  $\bar{\partial}^\mu \equiv \partial^\mu + \delta^\mu_0 \partial_0$ . Equation (31) determines  $g_1 = g_3 = g_4 = (D-2)$  and  $g_2 = (D-2)(D-1)$ . The trace of  $\mathcal{G}^{\mu\nu}$  is,

$$a^D \times \mathcal{G} \times a^{-(D-2)} \equiv \eta_{\mu\nu} \mathcal{G}^{\mu\nu} , \quad (37)$$

$$= -(D-2)a^D \left[ \frac{\nabla^2}{a^2} + (D-1) \frac{H}{a} \partial_0 + D(D-1)H^2 \right] \times a^{-(D-2)} . \quad (38)$$

### 3.2 The tensor projectors $\mathcal{F}^{\mu\nu\rho\sigma}$ and $\mathcal{G}^{\mu\nu\rho\sigma}$

Our technique for constructing transverse-traceless projectors is a variation of the one employed in the de Sitter invariant construction [9]. We begin by expanding the Weyl tensor of the conformally transformed metric,

$$\tilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu} \quad \Longrightarrow \quad \tilde{\mathcal{C}}_{\alpha\beta\gamma\delta} \equiv \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu} \times \kappa h_{\mu\nu} + O(\kappa^2 h^2) . \quad (39)$$

The second order differential operator  $\mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu}$  is,

$$\begin{aligned} \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu} = \mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu} - \frac{1}{D-2} \Big[ & \eta_{\alpha\gamma} \mathcal{D}_{\beta\delta}{}^{\mu\nu} - \eta_{\gamma\beta} \mathcal{D}_{\delta\alpha}{}^{\mu\nu} \\ & + \eta_{\beta\delta} \mathcal{D}_{\alpha\gamma}{}^{\mu\nu} - \eta_{\delta\alpha} \mathcal{D}_{\gamma\beta}{}^{\mu\nu} \Big] + \frac{(\eta_{\alpha\gamma} \eta_{\beta\delta} - \eta_{\alpha\delta} \eta_{\beta\gamma}) \mathcal{D}^{\mu\nu}}{(D-1)(D-2)} , \end{aligned} \quad (40)$$

where we can express  $\mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu}$  in terms of the Minkowski metric and the partial derivative operator,

$$\mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu} \equiv -\frac{1}{2} \left( \delta^{(\mu}_{\alpha} \delta^{\nu)}_{\gamma} \partial_{\beta} \partial_{\delta} - \delta^{(\mu}_{\gamma} \delta^{\nu)}_{\beta} \partial_{\delta} \partial_{\alpha} + \delta^{(\mu}_{\beta} \delta^{\nu)}_{\delta} \partial_{\alpha} \partial_{\gamma} - \delta^{(\mu}_{\delta} \delta^{\nu)}_{\alpha} \partial_{\gamma} \partial_{\beta} \right) , \quad (41)$$

$$\mathcal{D}_{\beta\delta}{}^{\mu\nu} \equiv \eta^{\alpha\gamma} \mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu} = -\frac{1}{2} \left( \eta^{\mu\nu} \partial_{\beta} \partial_{\delta} - 2 \partial^{(\mu} \delta^{\nu)}_{(\beta} \partial_{\delta)} + \delta^{(\mu}_{\beta} \delta^{\nu)}_{\delta} \partial^2 \right) , \quad (42)$$

$$\mathcal{D}^{\mu\nu} \equiv \eta^{\alpha\gamma} \eta^{\beta\delta} \mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu} = \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^2 . \quad (43)$$

Because the linearized Riemann tensor is invariant under linearized gauge transformations ( $\delta h_{\mu\nu} = -\partial_\mu \xi_\nu - \partial_\nu \xi_\mu$ ) the operator  $\mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu}$  and all its traces are transverse on the indices  $\mu$  and  $\nu$ . We also know that the linearized Weyl tensor vanishes for a conformal graviton field ( $h_{\mu\nu}(x) = \eta_{\mu\nu} \Omega^2(x)$ ), all of which implies that the operator  $\mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu}$  obeys two important identities,

$$\mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu} \times \eta_{\mu\nu} = 0 \quad , \quad \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu} \times \partial_\mu = 0 . \quad (44)$$

These identities mean we can define suitable transverse-traceless projectors by contracting  $\mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu}(x)$  times  $\mathcal{C}_{\kappa\lambda\theta\phi}{}^{\rho\sigma}(x')$  into any reflection invariant 8-index tensor. The choice made in the de Sitter invariant construction [9] was four products of the de Sitter invariant bi-tensor  $D^\alpha D'^\kappa y(x; x')$ , but that accounts for a large part of the complexity of the resulting representation. A far simpler — but noninvariant — representation will result from using products of  $\eta^{\alpha\kappa}$  and  $\overline{\eta}^{\alpha\kappa} \equiv \eta^{\alpha\kappa} + \delta^\alpha_0 \delta^\kappa_0$ ,

$$\mathcal{F}^{\mu\nu\rho\sigma} \equiv \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu}(x) \times \mathcal{C}_{\kappa\lambda\theta\phi}{}^{\rho\sigma}(x') \times \eta^{\alpha\kappa} \eta^{\beta\lambda} \eta^{\gamma\theta} \eta^{\delta\phi}, \quad (45)$$

$$\mathcal{G}^{\mu\nu\rho\sigma} \equiv \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu}(x) \times \mathcal{C}_{\kappa\lambda\theta\phi}{}^{\rho\sigma}(x') \times \overline{\eta}^{\alpha\kappa} \overline{\eta}^{\beta\lambda} \overline{\eta}^{\gamma\theta} \overline{\eta}^{\delta\phi}. \quad (46)$$

Explicit expressions for these operators are given in the Appendix.

## 4 Structure Functions for a MMC Scalar

Actual computations of the graviton self-energy will initially take the form of linear combinations of the basis tensors. The purpose of this section is first to explain generally how to reduce this initial primitive result to our form (30). We then apply this technique to work out the structure functions for the one loop contribution from a massless, minimally coupled scalar [9].

### 4.1 Finding the structure functions generally

Suppose a primitive result for the graviton self-energy  $-i[\mu\nu\Sigma^{\rho\sigma}](x; x')$  is known. Because this primitive form can be expressed in the form (30) we can reconstruct the four structure functions by picking off particularly simple tensor components. The procedure is first to trace on one index group, which makes the spin two contributions drop out, and then derive two linearly independent equations to reconstruct  $F_0(x; x')$  and  $G_0(x; x')$ . We then derive two linearly independent equations to reconstruct  $F_2(x; x')$  and  $G_2(x; x')$ . In what follows we will always assume  $i \neq j \neq k \neq i$ .

Tracing on  $\rho$  and  $\sigma$  gives,

$$-i[\mu\nu\Sigma^{\rho\sigma}](x; x') \times \eta_{\rho\sigma} = \mathcal{F}^{\mu\nu} \left( a'^D \mathcal{F}' \left[ \frac{F_0(x; x')}{a'^{D-2}} \right] \right) + \mathcal{G}^{\mu\nu} \left( a'^D \mathcal{G}' \left[ \frac{G_0(x; x')}{a'^{D-2}} \right] \right). \quad (47)$$

Our equations derive from the simple forms attained by the scalar projectors for  $\mu = 0, \nu = i$ ,

$$\mathcal{F}^{0i} = a^{D-2} \times \left[ -\partial_0 + aH \right] \partial_i \times a^{-(D-2)} , \quad (48)$$

$$\mathcal{G}^{0i} = a^{D-2} \times \left[ (D-2)aH \right] \partial_i \times a^{-(D-2)} , \quad (49)$$

and for  $\mu = j, \nu = k \neq j$ ,

$$\mathcal{F}^{jk} = a^{D-2} \times \partial_j \partial_k \times a^{-(D-2)} , \quad \mathcal{G}^{jk} = a^{D-2} \times \partial_j \partial_k \times a^{-(D-2)} . \quad (50)$$

By homogeneity and isotropy, these same index combinations for the self-energy must produce the same spatial derivatives. Because the self-energy is  $(aa')^2$  times a contravariant bi-tensor density, and  $\eta_{\rho\sigma} = g_{\rho\sigma}(x') \times a'^{-2}$ , it also makes sense to extract a factor of  $a^{D-2}a'^D$ ,

$$-i \left[ {}^{0i}\Sigma^{\rho\sigma} \right] (x; x') \times \eta_{\rho\sigma} \equiv a^{D-2} a'^D \partial_i S_1(x; x') , \quad (51)$$

$$-i \left[ {}^{jk}\Sigma^{\rho\sigma} \right] (x; x') \times \eta_{\rho\sigma} \equiv a^{D-2} a'^D \partial_j \partial_k S_2(x; x') . \quad (52)$$

Comparing relations (48-49) with (51) implies,

$$\left[ -\partial_0 + aH \right] \left( \mathcal{F}' \left[ \frac{F_0(x; x')}{(aa')^{D-2}} \right] \right) + (D-2)aH \left( \mathcal{G}' \left[ \frac{G_0(x; x')}{(aa')^{D-2}} \right] \right) = S_1(x; x') . \quad (53)$$

The second independent equation comes from comparing (50) with (52),

$$\mathcal{F}' \left[ \frac{F_0(x; x')}{(aa')^{D-2}} \right] + \mathcal{G}' \left[ \frac{G_0(x; x')}{(aa')^{D-2}} \right] = S_2(x; x') . \quad (54)$$

Given the two source functions  $S_1(x; x')$  and  $S_2(x; x')$ , we can obtain an equation for  $F_0(x; x')$  by subtracting  $(D-2)aH$  times (54) from (53),

$$\left[ \partial_0 + (D-3)aH \right] \left( \mathcal{F}' \left[ \frac{F_0(x; x')}{(aa')^{D-2}} \right] \right) = -S_1(x; x') + (D-2)aH S_2(x; x') . \quad (55)$$

The solution can be expressed as an indefinite integral,

$$\mathcal{F}' \left[ \frac{F_0(x; x')}{(aa')^{D-2}} \right] = a^{-(D-3)} \int d\eta a^{D-3} \left[ -S_1(x; x') + (D-2)aH S_2(x; x') \right] . \quad (56)$$

The comparable relation for  $G_0(x; x')$  comes from subtracting (56) from (54),

$$\mathcal{G}' \left[ \frac{G_0(x; x')}{(aa')^{D-2}} \right] = S_2(x; x') + a^{-(D-3)} \int d\eta a^{D-3} \left[ S_1(x; x') - (D-2)aH S_2(x; x') \right]. \quad (57)$$

We can recover the structure function  $F_0(x; x')$  from expression (56) by employing the Green's function for  $\mathcal{F}'$ ,

$$\mathcal{F}' = -(D-1) \left[ \square' + DH^2 \right]. \quad (58)$$

This Greens' function is proportional to the scalar propagator for a tachyonic mass of  $M^2 = -DH^2$ , and its specialization to a de Sitter invariant source was derived in [9]. The structure function  $G_0(x; x')$  comes from integrating expression (57) against the Green's function for  $\mathcal{G}'$ ,

$$\mathcal{G}' = -\frac{(D-2)}{a'^2} \left[ \nabla'^2 + (D-1)a'H\partial'_0 + D(D-1)a'^2 H^2 \right]. \quad (59)$$

The key to determining the spin two structure functions is a set of four identities for the projectors  $\mathcal{F}^{0ijk}$  (with  $i \neq j \neq k \neq i$ ) which can be derived for (after using homogeneity to reflect spatial derivatives  $\partial'_i = -\partial_i$ ) from the explicit forms given in the Appendix,

$$\mathcal{F}^{0ijk} = \frac{(D-3)}{(D-1)(D-2)} \left[ (D-1)\partial'_0 + \partial_0 \right] \partial_i \partial_j \partial_k, \quad (60)$$

$$\mathcal{F}^{jk0i} = \frac{(D-3)}{(D-1)(D-2)} \left[ -\partial'_0 - (D-1)\partial_0 \right] \partial_i \partial_j \partial_k, \quad (61)$$

$$\mathcal{G}^{0ijk} = \frac{(D-3)}{(D-1)(D-2)^2} \times \partial_0 \partial_i \partial_j \partial_k, \quad (62)$$

$$\mathcal{G}^{jk0i} = \frac{(D-3)}{(D-1)(D-2)^2} \times -\partial'_0 \partial_i \partial_j \partial_k. \quad (63)$$

Assuming the spin zero structure functions are known we can reconstruct the spin two structure functions from sums and differences of the  $0ijk$  and  $jk0i$  components. From homogeneity and isotropy, and a judicious guess for the scale factors, we can express these components as,

$$-i \left[ {}^{0i}\Sigma^{jk} \right] (x; x') = (aa')^{D-2} \partial_i \partial_j \partial_k S_3(x; x'), \quad (64)$$

$$-i \left[ {}^{jk}\Sigma^{0i} \right] (x; x') = (aa')^{D-2} \partial_i \partial_j \partial_k S_4(x; x'). \quad (65)$$

Combining these relations with (60-63), and expressions (48-50) allows us to derive first order differential equations for the spin two structure functions,

$$\frac{(D-3)(\partial'_0 - \partial_0)}{(D-1)(D-2)^2} \left[ (D-2)^2 F_2(x; x') - G_2(x; x') \right] = (aa')^{D-2} [S_3 + S_4] \\ - \left[ \partial'_0 - \partial_0 + (D-1)(a-a')H \right] F_0 - (D-2)(a-a')HG_0, \quad (66)$$

$$\frac{(D-3)(\partial'_0 + \partial_0)}{(D-1)(D-2)^2} \left[ D(D-2)F_2(x; x') + G_2(x; x') \right] = (aa')^{D-2} [S_3 - S_4] \\ + \left[ \partial'_0 + \partial_0 - (D-1)(a+a')H \right] F_0 - (D-2)(a+a')HG_0. \quad (67)$$

Because equations (66) and (67) determine different linear combinations of  $F_2(x; x')$  and  $G_2(x; x')$  we can recover both of the spin two structure functions.

## 4.2 Primitive contribution from a MMC scalar

The contribution to the graviton self-energy from a loop of massless, minimally coupled (MMC) scalars can be expressed as a linear combination of the de Sitter invariant basis tensors [9, 21],

$$-i \left[ {}^{\mu\nu} \Sigma^{\rho\sigma} \right] = (aa')^{D+2} \left\{ D^\mu D'^{\rho} y D'^{\sigma} D^\nu y \alpha + D^{(\mu} y D^{\nu)} D'^{\rho} y D'^{\sigma} y \beta \right. \\ \left. + D^\mu y D^\nu y D'^{\rho} y D'^{\sigma} y \gamma + H^4 g^{\mu\nu} g'^{\rho\sigma} \delta + H^2 \left[ g^{\mu\nu} D'^{\rho} y D'^{\sigma} y + (\text{refl.}) \right] \epsilon \right\}. \quad (68)$$

The combination coefficients are functions of the de Sitter length function  $y(x; x')$  and derivatives of the function  $A(y)$  [9],

$$\alpha(y) = -\frac{\kappa^2}{2} [A'(y)]^2, \quad \beta(y) = -\kappa^2 A'(y) A''(y), \quad \gamma(y) = -\frac{\kappa^2}{2} [A''(y)]^2, \quad (69)$$

$$\delta(y) = -\frac{\kappa^2}{8} \left[ (4y-y^2)^2 (A'')^2 + 2(2-y)(4y-y^2) A' A'' + \left[ 4(D-4) - (4y-y^2) \right] (A')^2 \right], \quad (70)$$

$$\epsilon(y) = \frac{\kappa^2}{4} \left[ (4y-y^2) [A''(y)]^2 + 2(2-y) A'(y) A''(y) - [A'(y)]^2 \right]. \quad (71)$$

The function  $A(y)$  is the de Sitter invariant part of the MMC scalar propagator  $i\Delta(x; x') = A(y) + k \ln(aa')$  [22],

$$A(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{\Gamma(\frac{D}{2})}{\frac{D}{2}-1} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2} \left(\frac{4}{y}\right)^{\frac{D}{2}-2} + \text{constant} \right. \\ \left. + \sum_{n=1}^{\infty} \left[ \frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right] \right\}, \quad (72)$$

Note that the infinite series on the second line of (72) vanishes in  $D = 4$ , so these terms only need to be retained when they multiply a divergent term. It is also worth noting that  $A(y)$  obeys the equation,

$$(4y-y^2)A''(y) + D(2-y)A'(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D)}{\Gamma(\frac{D}{2})} \equiv (D-1)k. \quad (73)$$

### 4.3 Spin zero structure functions for a MMC scalar

The trace of expression (68) produces one term proportional to  $g^{\mu\nu}$  and another proportional to  $D^\mu y D^\nu y$ ,

$$-i \left[ {}^{\mu\nu}\Sigma^{\rho\sigma} \right] (x; x') \times \eta_{\rho\sigma} = a^2 H^2 (aa')^D \left\{ H^2 g^{\mu\nu} \left[ 4\alpha + D\delta + (4y-y^2)\epsilon \right] \right. \\ \left. + D^\mu y D^\nu y \left[ -\alpha + (2-y)\beta + (4y-y^2)\gamma + D\epsilon \right] \right\}. \quad (74)$$

Now recall from (7) that the components we need of  $D^\mu y D^\nu y$  are,

$$D^0 y D^i y = a^{-4} \times -aH \left( y-2+2\frac{a'}{a} \right) \partial_i y \quad , \quad D^j y D^k y = a^{-4} \times \partial_j y \partial_k y. \quad (75)$$

The two scalar sources follow from comparison with expressions (51-52),

$$S_1(x; x') = -aH^3 I \left[ (y-2)F'' \right] - 2a'H^3 F'(y) = H^2 \left( -\partial_0 + aH \right) F(y), \quad (76)$$

$$S_2(x; x') = H^2 F(y), \quad (77)$$

where the function  $F(y)$  is a double indefinite integral,<sup>1</sup>

$$F(y) \equiv I^2 \left[ -\alpha + (2-y)\beta + (4y-y^2)\gamma + D\epsilon \right]. \quad (78)$$

---

<sup>1</sup>We define the indefinite integral of a function  $f(y)$  as  $I[f](y) \equiv \int^y dz f(z)$ .

Substituting (76-77) into expressions (56) and (57) gives the spin zero structure functions,

$$F_0(x; x') = -\frac{(aa')^{D-2}}{D-1} \left( \frac{H^2}{\square + DH^2} \right) F(y) \quad , \quad G_0(x; x') = 0 \quad . \quad (79)$$

In retrospect we can observe that the vanishing of  $G_0(x; x')$  is a consequence of the fact that our first spin zero term  $\mathcal{F}^{\mu\nu} \times \mathcal{F}'^{\rho\sigma} [F_0(x; x')]$  is just a conformal transformation of the single spin zero contribution in the de Sitter invariant construction [9]. The second spin zero structure function  $G_0(x; x')$  must vanish whenever the graviton self-energy is de Sitter invariant.

In principle, we could read off the fully renormalized result for  $F_0(x; x')$  as  $(aa')^{D-2} \times \mathcal{F}_{1R}(y)$ , from equation (234) of [9]. However, there are some subtleties to inverting  $\square + DH^2$  that were not previously understood, and our current approach has the significant simplification of only requiring a single inversion rather than two [9]. We will therefore carry out the derivation.

Substituting expression (72) into (69-71), and then into (78) gives the following expansion for the function  $F(y)$ ,

$$F(y) = \frac{\kappa^2 H^{2D-4} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \left\{ \frac{(D-2)^2}{16(D-1)} \left( \frac{4}{y} \right)^{D-1} + \frac{(D^3 - 5D^2 + 6D - 4)}{16(D-1)} \left( \frac{4}{y} \right)^{D-2} + \dots \right\}. \quad (80)$$

The neglected terms in this and subsequent expansions have the twin properties that they make integrable contributions to the structure functions, and they vanish in  $D = 4$  dimensions. Inverting  $(\square/H^2 + D)$  on  $F(y)$  amounts to solving the differential equation,

$$\left[ \frac{\square}{H^2} + D \right] f(x; x') = F(y(x; x')) \quad . \quad (81)$$

The first step is to expand  $f(x; x')$  so as to absorb the leading terms of  $F(y)$  in expression (80),

$$f(x; x') = \frac{\kappa^2 H^{2D-4} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \left\{ \frac{1}{8(D-1)} \left( \frac{4}{y} \right)^{D-2} + \frac{D(D^2 - 5D + 2)}{8(D-4)(D-3)(D-1)} \left( \frac{4}{y} \right)^{D-3} + \Delta f_0(x; x') \right\}, \quad (82)$$



where the remainder  $\Delta f_0(x; x')$  obeys,

$$\left[ \frac{\square}{H^2} + D \right] \Delta f_0(x; x') = -\frac{3(D-2)D(D^2-5D+2)}{8(D-4)(D-3)(D-1)} \left( \frac{4}{y} \right)^{D-3}. \quad (83)$$

Note that we could set  $D = 4$  for the  $(4/y)^{D-3}$  terms of expressions (82) and (83) were it not for the multiplicative factors of  $1/(D-4)$ . We can localize the divergence by adding zero based on the identity,<sup>2</sup>

$$\left[ \frac{\square}{H^2} + D \right] \left( \frac{4}{y} \right)^{\frac{D}{2}-1} = \frac{D(D+2)}{4} \left( \frac{4}{y} \right)^{\frac{D}{2}-1} + \frac{(4\pi)^{\frac{D}{2}} i \delta^D(x-x')}{(Ha)^D \Gamma(\frac{D}{2}-1)}. \quad (84)$$

The revised expansion becomes,

$$f(x; x') = \frac{\kappa^2 H^{2D-4} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \left\{ \frac{1}{8(D-1)} \left( \frac{4}{y} \right)^{D-2} + \frac{D(D^2-5D+2)}{8(D-4)(D-3)(D-1)} \left[ \left( \frac{4}{y} \right)^{D-3} - \left( \frac{4}{y} \right)^{\frac{D}{2}-1} \right] + \Delta f_1(x; x') \right\}, \quad (85)$$

where the new remainder  $\Delta f_1(x; x')$  obeys,

$$\left[ \frac{\square}{H^2} + D \right] \Delta f_1(x; x') = \frac{D(D^2-5D+2)}{8(D-4)(D-3)(D-1)} \times \frac{(4\pi)^{\frac{D}{2}} i \delta^D(x-x')}{(Ha)^D \Gamma(\frac{D}{2}-1)} - \frac{3(D-2)D(D^2-5D+2)}{8(D-4)(D-3)(D-1)} \left[ \left( \frac{4}{y} \right)^{D-3} - \frac{D(D+2)}{12(D-2)} \left( \frac{4}{y} \right)^{\frac{D}{2}-1} \right]. \quad (86)$$

The term proportional to the delta function in expression (86) can be absorbed into the counterterm  $\Delta \mathcal{L}_3 = c_3 H^2 [R - (D-1)(D-2)H^2] \sqrt{-g}$  of Ref. [9], see especially equations (115) and (228) of that work. When this is done we can set  $D = 4$  in all but the first term of (85). The resulting, partially renormalized expansion is,

$$f(x; x') = \frac{\kappa^2 H^{2D-4} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \left\{ \frac{1}{8(D-1)} \left( \frac{4}{y} \right)^{D-2} + \frac{2 \ln(\frac{y}{4})}{3y} + \Delta f_2(x; x') \right\}, \quad (87)$$

---

<sup>2</sup>In dimensional regularization it is easy to show that one only gets delta functions from differentiating  $1/y^{\frac{D}{2}+N}$ , where  $N = -1, 0, 1, 2, \dots$ . Even terms which become identical in four dimensions — for example,  $1/y^{D+N-2}$  — do not produce delta functions [22].

where the renormalized remainder  $\Delta f_2(x; x')$  obeys,

$$\left[ \frac{\square}{H^2} + 4 \right] \Delta f_2(x; x') = - \left[ \frac{4 \ln\left(\frac{y}{4}\right) - \frac{2}{3}}{y} \right] \equiv \Delta F_2(y) . \quad (88)$$

The next step is to derive a formal solution to (88) using the de Sitter invariant Green's function that can be constructed from the two homogeneous solutions [9],

$$\phi_1(y) = 2 - y , \quad (89)$$

$$\phi_2(y) = -\frac{2}{y} - \frac{2}{4-y} + \frac{3}{2}(2-y) \left[ \ln\left(\frac{y}{4}\right) - \ln\left(1 - \frac{y}{4}\right) \right] + 6 . \quad (90)$$

Although (89) obeys  $(\square + 4H^2)\phi_1(y) = 0$ , acting  $(\square + 4H^2)$  on expression (90) actually produces delta functions at the origin and at the antipodal point. (This is the subtlety that was not understood in the original construction [9].) We can nonetheless derive a homogeneous and isotropic solution for  $\Delta f_2(x; x')$  by a process of employing the formal Green's function,

$$\mathcal{G}(y; y') = \theta(y - y') \left[ \phi_2(y)\phi_1(y') - \phi_1(y)\phi_2(y') \right] \mathcal{W}(y') , \quad \mathcal{W}(y) \equiv \frac{(4y - y^2)}{64} , \quad (91)$$

and then subtracting off the unwanted pole terms.

The formal, de Sitter invariant solution to (88) can be expressed using the indefinite integral operation  $I[\dots]$ ,

$$\begin{aligned} \phi_2(y) I \left[ \phi_1 \mathcal{W} \Delta F_2 \right] (y) - \phi_1(y) I \left[ \phi_2 \mathcal{W} \Delta F_2 \right] (y) &= \frac{-2}{4-y} - \frac{29}{3} + \frac{41}{6} y \\ &+ \left[ \frac{\frac{2}{3}}{4-y} + 1 - \frac{3}{2} y \right] \ln\left(\frac{y}{4}\right) - (2-y) \left[ \frac{3}{2} \ln\left(1 - \frac{y}{4}\right) + \frac{1}{2} \Psi(y) \right] , \end{aligned} \quad (92)$$

where the function  $\Psi(y)$  will appear in all the structure functions,

$$\Psi(y) \equiv \frac{1}{2} \ln^2\left(\frac{y}{4}\right) - \ln\left(1 - \frac{y}{4}\right) \ln\left(\frac{y}{4}\right) - \text{Li}_2\left(\frac{y}{4}\right) . \quad (93)$$

Expression (92) is only a formal solution to (88) because of the pole it has at the antipodal point of  $y = 4$ . We can eliminate this pole by adding  $\frac{41}{6}\phi_1(y) - \phi_2(y)$ ,

$$\Delta f_2(x; x') = \frac{2}{y} - 2 + \left[ \frac{\frac{2}{3}}{4-y} - 2 \right] \ln\left(\frac{y}{4}\right) - \left( \frac{2-y}{2} \right) \Psi(y) . \quad (94)$$

Substituting expressions (94) and (87) into (79) allows us to at length express the scalar structure function as  $F_0(x; x') = (aa')^{D-2}\Phi(y)$  where,

$$\Phi(y) = \frac{\kappa^2 H^{2D-4} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \left\{ -\frac{1}{8(D-1)^2} \left(\frac{4}{y}\right)^{D-2} - \frac{2}{3y} - \frac{2}{9} \left[ \frac{1}{y} - 3 + \frac{1}{4-y} \right] \ln\left(\frac{y}{4}\right) + \frac{2}{3} + \left(\frac{2-y}{6}\right) \Psi(y) \right\}. \quad (95)$$

The most singular part of  $F_0(x; x')$  can be partially integrated to isolate the remaining ultraviolet divergence,

$$\begin{aligned} & \frac{\kappa^2 (H^2 aa')^{D-2} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \times -\frac{1}{8(D-1)^2} \left(\frac{4}{y}\right)^{D-2} \\ &= \frac{\kappa^2 \Gamma^2(\frac{D}{2})}{16\pi^D} \times -\frac{\partial^2}{16(D-4)(D-3)(D-1)^2} \left[ \frac{1}{\Delta x^{2D-6}} \right], \end{aligned} \quad (96)$$

$$\begin{aligned} &= \frac{\kappa^2 \Gamma^2(\frac{D}{2})}{16\pi^D} \frac{-1}{16(D-4)(D-3)(D-1)^2} \\ &\quad \times \left\{ \partial^2 \left[ \frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}} \right] + \frac{4\pi^{\frac{D}{2}} \mu^{D-4} i \delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} \right\}, \end{aligned} \quad (97)$$

$$\longrightarrow \frac{\kappa^2}{(4\pi)^4} \frac{\partial^2}{18} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] - \frac{\kappa^2 \mu^{D-4} \Gamma(\frac{D}{2})}{128\pi^{\frac{D}{2}}} \frac{(D-2) i \delta^D(x-x')}{(D-4)(D-3)(D-1)^2}. \quad (98)$$

The divergence in expression (98) can be absorbed into the counterterm  $\Delta\mathcal{L}_1 = c_1[R - D(D-1)H^2]^2 \sqrt{-g}$  of Ref. [9], see especially equations (113) and (227) of that work.<sup>3</sup> Our final result for the renormalized scalar structure function is therefore,

$$\begin{aligned} F_{0R}(x; x') = \frac{\kappa^2 (aa' H^2)^2}{2304\pi^4} & \left\{ \frac{\partial^2}{2(aa' H^2)^2} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] - \frac{6}{y} + 6 \right. \\ & \left. + \left[ -\frac{2}{y} + 6 - \frac{2}{4-y} \right] \ln\left(\frac{y}{4}\right) + \frac{3}{2}(2-y)\Psi(y) \right\}. \end{aligned} \quad (99)$$

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<sup>3</sup>Complete agreement with Ref. [9] requires our renormalization scale to be  $\mu = \frac{1}{2}H$ .

#### 4.4 Spin two structure functions for a MMC scalar

In addition to the identities (75) and their reflections, extracting the sources  $S_3(x; x')$  and  $S_4(x; x')$  from the  $0ijk$  and  $jk0i$  components requires an identity we can infer from (8) for the mixed derivative,

$$D^0 D'^j y = (aa')^{-2} \times aH \partial_j y \quad , \quad D^i D'^0 y = (aa')^{-2} \times -a' H \partial_i y . \quad (100)$$

Applying these identities to the desired components of (68) gives,

$$\begin{aligned} -i \left[ {}^{0i} \Sigma^{jk} \right] (x; x') &= (aa')^{D+2} \left\{ \frac{1}{2} D^i y D^0 D'^{(j} y D'^{k)} y \beta + D^0 y D^i y D'^j y D'^k y \gamma \right\} , \\ &= (aa')^{D-2} \partial_i \partial_j \partial_k \left\{ -\frac{1}{2} a H I^3[\beta] - a H I^3[(y-2)\gamma] - 2a' H I^3[\gamma] \right\} , \end{aligned} \quad (101)$$

$$\begin{aligned} -i \left[ {}^{jk} \Sigma^{0i} \right] (x; x') &= (aa')^{D+2} \left\{ \frac{1}{2} D^{(j} y D^k y D'^0 y D'^i y \beta + D^j y D^k y D'^0 y D'^i y \gamma \right\} , \\ &= (aa')^{D-2} \partial_i \partial_j \partial_k \left\{ \frac{1}{2} a' H I^3[\beta] + a' H I^3[(y-2)\gamma] + 2a H I^3[\gamma] \right\} . \end{aligned} \quad (102)$$

Use of the partial integration identity  $I^3[(y-2)\gamma] = (y-2)I^3[\gamma] - 3I^4[\gamma]$  and comparison with expressions (64-65) implies,

$$S_3(x; x') = -\frac{1}{2} a H I^3[\beta] + \left( -\partial_0 + 3aH \right) I^4[\gamma] , \quad (103)$$

$$S_4(x; x') = +\frac{1}{2} a' H I^3[\beta] + \left( \partial'_0 - 3a'H \right) I^4[\gamma] . \quad (104)$$

Substituting expressions (103-104) into (66-67), and setting the spin zero structure functions to  $F_0(x; x') \equiv (aa')^{D-2} \times \Phi(y)$  and  $G_0(x; x') = 0$ , gives equations for  $F_2(x; x')$  and  $G_2(x; x')$ ,

$$\begin{aligned} (\partial'_0 - \partial_0) \left\{ \left( \frac{D-3}{D-1} \right) F_2 - \frac{(D-3) G_2}{(D-1)(D-2)^2} - (aa')^{D-2} I^4[\gamma] + F_0 \right\} \\ = (a-a')H \times (aa')^{D-2} \left\{ -\frac{1}{2} I^3[\beta] + (D+1)I^4[\gamma] - (D-1)\Phi \right\} , \end{aligned} \quad (105)$$

$$\begin{aligned} (\partial'_0 + \partial_0) \left\{ \frac{D(D-3)}{(D-1)(D-2)} F_2 + \frac{(D-3) G_2}{(D-1)(D-2)^2} + (aa')^{D-2} I^4[\gamma] - F_0 \right\} \\ = (a+a')H \times (aa')^{D-2} \left\{ -\frac{1}{2} I^3[\beta] + (D+1)I^4[\gamma] - (D-1)\Phi \right\} , \end{aligned} \quad (106)$$

To solve equation (105), consider acting  $\partial'_0 - \partial_0$  on  $(aa')^{D-2} \times f(y)$ ,

$$(\partial'_0 - \partial_0) \left[ (aa')^{D-2} f(y) \right] = (a-a') H(aa')^{D-2} \left\{ (4-y) f'(y) - (D-2) f(y) \right\}. \quad (107)$$

The solution to (105) can therefore be expressed as an indefinite integral,

$$\begin{aligned} & \left( \frac{D-3}{D-1} \right) F_2 - \frac{(D-3) G_2}{(D-1)(D-2)^2} - (aa')^{D-2} I^4[\gamma] + F_0 = \frac{(aa')^{D-2}}{(4-y)^{D-2}} \\ & \times \left\{ I \left[ (4-y)^{D-3} \left\{ -\frac{1}{2} I^3[\beta] + (D+1) I^4[\gamma] - (D-1) \Phi \right\} \right] + K_1 \right\}, \end{aligned} \quad (108)$$

where the integration constant  $K_1$  is chosen so that there is no singularity at the antipodal point  $y = 4$ . Of course the same considerations apply for equation (106),

$$(\partial'_0 + \partial_0) \left[ (aa')^{D-2} f(y) \right] = (a+a') H(aa')^{D-2} \left\{ y f'(y) + (D-2) f(y) \right\}. \quad (109)$$

The solution to (106) is therefore,

$$\begin{aligned} & \frac{D(D-3)}{(D-1)(D-2)} F_2 + \frac{(D-3) G_2}{(D-1)(D-2)^2} + (aa')^{D-2} I^4[\gamma] - F_0 \\ & = \frac{(aa')^{D-2}}{y^{D-2}} \left\{ I \left[ y^{D-3} \left\{ -\frac{1}{2} I^3[\beta] + (D+1) I^4[\gamma] - (D-1) \Phi \right\} \right] + K_2 \right\}, \end{aligned} \quad (110)$$

where the integration constant  $K_2$  is chosen to prevent  $G_2(x; x')$  from having any term proportional to  $1/y^{D-2}$ .

We have already given the expansion (95) for the de Sitter invariant part  $\Phi(y)$  of the scalar structure function. Substituting expression (72) into (69) gives the additional expansions we require,

$$I^3[\beta] = \frac{\kappa^2 H^{2D-4} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \left\{ \frac{-(\frac{4}{y})^{D-2}}{2(D-1)(D-2)} - \frac{4}{y} + 2 \ln\left(\frac{y}{4}\right) + \dots \right\}, \quad (111)$$

$$I^4[\gamma] = \frac{\kappa^2 H^{2D-4} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \left\{ \frac{-D(\frac{4}{y})^{D-2}}{8(D+1)(D-1)(D-2)} - \frac{2}{3y} + \frac{1}{3} \ln\left(\frac{y}{4}\right) + \dots \right\}. \quad (112)$$

As before, the neglected terms have the twin properties of making integrable contributions to the structure functions and vanishing in  $D = 4$  dimensions.

Substituting these expansions into expressions (108) and (110) gives the following expansions for the tensor structure functions,

$$F_2(x; x') = \frac{\kappa^2 (H^2 a a')^{D-2} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \left\{ \frac{-\left(\frac{4}{y}\right)^{D-2}}{4(D+1)(D-1)(D-2)(D-3)} + \frac{2}{3} \left[ \frac{1}{y} - \frac{1}{4-y} \right] \ln\left(\frac{y}{4}\right) - \frac{1}{3} \Psi(y) \right\}, \quad (113)$$

$$G_2(x; x') = \frac{\kappa^2 (H^2 a a')^2}{(4\pi)^4} \left\{ -2 + \frac{8}{3} \frac{\ln(\frac{y}{4})}{(4-y)} + \frac{2}{3} \Psi(y) \right\}. \quad (114)$$

The second tensor structure function  $G_2(x; x')$  is ultraviolet finite but the first term of  $F_2(x; x')$  requires the same treatment as the most singular part of  $F_0(x; x')$ . The steps are the same as led to expression (98) so we merely give the result,

$$\begin{aligned} & \frac{\kappa^2 (H^2 a a')^{D-2} \Gamma^2(\frac{D}{2})}{(4\pi)^D} \times -\frac{1}{4(D+1)(D-1)(D-2)(D-3)} \left(\frac{4}{y}\right)^{D-2} \\ & \longrightarrow \frac{\kappa^2}{(4\pi)^4} \frac{\partial^2}{30} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] - \frac{\kappa^2 \mu^{D-4} \Gamma(\frac{D}{2})}{64\pi^{\frac{D}{2}}} \frac{i\delta^D(x-x')}{(D-4)(D-3)^2(D-1)(D+1)}. \end{aligned} \quad (115)$$

The divergent part of expression (115) can be absorbed into the counterterm  $\Delta\mathcal{L}_2 = c_2 C^{\alpha\beta\gamma\delta} C_{\alpha\beta\gamma\delta} \sqrt{-g}$  of Ref. [9], see equations (114) and (237) of that work. Hence our final renormalized result for the first tensor structure function is,

$$F_{2R}(x; x') = \frac{\kappa^2 (H^2 a a')^2}{(4\pi)^4} \left\{ \frac{\partial^2}{30(H^2 a a')^2} \left[ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \frac{2}{3} \left[ \frac{1}{y} - \frac{1}{4-y} \right] \ln\left(\frac{y}{4}\right) - \frac{1}{3} \Psi(y) \right\}. \quad (116)$$

As desired, expressions (116) and (114) are significantly simpler than the form which pertains for the manifestly de Sitter invariant construction of Ref. [9].

## 5 Quantum Corrections to Gravitons

The point of this section is to re-examine the conclusion [10] of the de Sitter invariant analysis that the ensemble of MMC scalars produced during inflation has no effect on dynamical gravitons at one loop. We begin by explaining how to perturbatively formulate the effective field equations of the Schwinger-Keldysh formalism [23, 24]. Then we specialize to quantum correcting the mode functions of plane wave gravitons.

### 5.1 Perturbative effective field equations

The linearized effective field equation for our graviton field (18) is,

$$\partial_\alpha \left[ a^2 \mathcal{L}^{\mu\nu\rho\sigma\alpha\beta} \partial_\beta h_{\rho\sigma}(x) \right] - \int d^4x' \left[ {}^{\mu\nu}\Sigma^{\rho\sigma} \right](x; x') h_{\rho\sigma}(x') = -\frac{\kappa a^2}{2} \eta^{\mu\rho} \eta^{\nu\sigma} T_{\rho\sigma}(x) . \quad (117)$$

Here  $T_{\rho\sigma}(x)$  is the stress-energy tensor and  $\mathcal{L}^{\mu\nu\rho\sigma\alpha\beta}$  is,

$$\begin{aligned} \mathcal{L}^{\mu\nu\rho\sigma\alpha\beta} = & \frac{1}{2} \eta^{\alpha\beta} \left[ \eta^{\mu(\rho} \eta^{\sigma)\nu} - \eta^{\mu\nu} \eta^{\rho\sigma} \right] \\ & + \frac{1}{2} \eta^{\mu\nu} \eta^{\rho(\alpha} \eta^{\beta)\sigma} + \frac{1}{2} \eta^{\rho\sigma} \eta^{\mu(\alpha} \eta^{\beta)\nu} - \eta^{\alpha(\rho} \eta^{\sigma)(\mu} \eta^{\nu)(\beta)} \end{aligned} \quad (118)$$

With  $T_{\rho\sigma} = 0$  one can use equation (117) to understand how inflationary particles affect the propagation of dynamical gravitons. By setting  $T_{\rho\sigma} \neq 0$  one can study how inflation affects the force of gravity.

It is useful to re-express (117) in terms of the four structure functions  $F_0(x; x')$ ,  $G_0(x; x')$ ,  $F_2(x; x')$  and  $G_2(x; x')$  in our representation (30) of the graviton self-energy  $[{}^{\mu\nu}\Sigma^{\rho\sigma}](x; x')$ . Recall that each term is the product of primed and unprimed, second order differential operators acting on one of the structure functions. We can extract the unprimed differential operator from the integration over  $x'^\mu$ , and partially integrate the primed differential operator to act on the graviton field  $h_{\rho\sigma}(x')$ . Carrying this out for the  $F_2$  structure function gives,

$$\begin{aligned} \int d^4x' \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu} \times \mathcal{C}'_{\kappa\lambda\theta\phi}{}^{\rho\sigma} \left[ \eta^{\alpha\kappa} \eta^{\beta\lambda} \eta^{\gamma\theta} \eta^{\delta\phi} i F_2(x; x') \right] \times h_{\rho\sigma}(x') \\ = \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu} \int d^4x' i F_2(x; x') \times \tilde{C}_{\text{lin}}^{\alpha\beta\gamma\delta}(x') , \end{aligned} \quad (119)$$

$$= -2\partial_\alpha \partial_\beta \int d^4x' i F_2(x; x') \times \tilde{C}_{\text{lin}}^{\mu\alpha\nu\beta}(x') . \quad (120)$$

Here  $\kappa \tilde{C}_{\text{lin}}^{\alpha\beta\gamma\delta}(x)$  is the linearized Weyl tensor of the conformally rescaled metric  $\tilde{g}_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$ . The transition from (119) to (120) is justified by the tracelessness of the Weyl tensor and by its algebraic symmetries.

With the various projectors extracted or partially integrated the effective field equation (117) takes the form,

$$\begin{aligned} \partial_\alpha \left[ a^2 \mathcal{L}^{\mu\nu\rho\sigma\alpha\beta} \partial_\beta h_{\rho\sigma}(x) \right] &= -\frac{\kappa a^2}{2} \eta^{\mu\rho} \eta^{\nu\sigma} T_{\rho\sigma}(x) \\ &+ \mathcal{F}^{\mu\nu} \int d^4 x' i F_0(x; x') \mathcal{R}(x') + \mathcal{G}^{\mu\nu} \int d^4 x' i G_0(x; x') \mathcal{S}(x') \\ &- 2 \partial_\alpha \partial_\beta \int d^4 x' \left[ i F_2(x; x') \tilde{C}_{\text{lin}}^{\mu\alpha\nu\beta}(x') + i G_2(x; x') \overline{\tilde{C}_{\text{lin}}}^{\mu\alpha\nu\beta}(x') \right] \\ &+ \left[ \eta^{\mu\nu} \partial_k \partial_\ell - 2 \delta_{(k}^{\mu} \partial^{\nu)} \partial_\ell + \delta_k^{(\mu} \delta_\ell^{\nu)} \partial^2 \right] \int d^4 x' i G_2(x; x') \tilde{C}_{\text{lin}}^{0k0\ell}(x') . \end{aligned} \quad (121)$$

We remind the reader that the spin zero projectors  $\mathcal{F}^{\mu\nu}$  and  $\mathcal{G}^{\mu\nu}$  were defined in expressions (33) and (36), respectively, although they should be specialized to  $D = 4$  dimensions here. The quantity  $\mathcal{R}$  is  $\kappa^{-1}$  times the linear part of the ( $D = 4$ ) Ricci scalar,

$$\mathcal{R}(x) \equiv \partial^\rho \partial^\sigma h_{\rho\sigma} - 6aH \partial^\rho h_{0\rho} + 12a^2 H^2 h_{00} - \left[ \partial^2 - 3aH \partial_0 \right] h . \quad (122)$$

The “essentially spatial” part of this is,

$$\mathcal{S}(x) \equiv \partial_k \partial_\ell h_{k\ell} - 4aH \partial_k h_{0k} + 6a^2 H^2 h_{00} - \left[ \nabla^2 - 2aH \partial_0 \right] h_{kk} . \quad (123)$$

The symbol  $\overline{\tilde{C}_{\text{lin}}}^{\alpha\beta\gamma\delta}$  stands for the purely spatial components of the linearized Weyl tensor of the conformally rescaled metric.

Equation (121) is general, but its use is limited because we will never possess more than the lowest loop results for the various structure functions. The only valid solution is to regard  $h_{\mu\nu}(x)$ , and the structure functions, as the sum of results at different loop orders,

$$h_{\mu\nu}(x) = h_{\mu\nu}^0(x) + h_{\mu\nu}^1(x) + h_{\mu\nu}^2(x) + \dots \quad (124)$$

$$F_{0,2}(x; x') = 0 + F_{0,2}^1(x; x') + F_{0,2}^2(x; x') + \dots \quad (125)$$

$$G_{0,2}(x; x') = 0 + G_{0,2}^1(x; x') + G_{0,2}^2(x; x') + \dots \quad (126)$$



Unless the stress tensor includes loop corrections from the 1PI 1-point function, we regard it as 0th order,

$$\partial_\alpha \left[ a^2 \mathcal{L}^{\mu\nu\rho\sigma\alpha\beta} \partial_\beta h_{\rho\sigma}^0(x) \right] = -\frac{\kappa a^2}{2} \eta^{\mu\rho} \eta^{\nu\sigma} T_{\rho\sigma}(x) . \quad (127)$$

The resulting zero loop field  $h_{\mu\nu}^0$  then combines with the one loop structure functions in (121) to provide sources for the one loop field  $h_{\mu\nu}^1$ ,

$$\begin{aligned} & \partial_\alpha \left[ a^2 \mathcal{L}^{\mu\nu\rho\sigma\alpha\beta} \partial_\beta h_{\rho\sigma}^1(x) \right] \\ &= \mathcal{F}^{\mu\nu} \int d^4x' iF_0^1(x; x') \mathcal{R}^0(x') + \mathcal{G}^{\mu\nu} \int d^4x' iG_0^1(x; x') \mathcal{S}^0(x') \\ & \quad - 2\partial_\alpha \partial_\beta \int d^4x' \left[ iF_2^1(x; x') \tilde{C}_{\text{lin}0}^{\mu\alpha\nu\beta}(x') + iG_2^1(x; x') \tilde{\overline{C}}_{\text{lin}0}^{\mu\alpha\nu\beta}(x') \right] \\ & \quad + \left[ \eta^{\mu\nu} \partial_k \partial_\ell - 2\delta_{(k}^{(\mu} \partial^{\nu)} \partial_\ell + \delta_k^{(\mu} \delta_\ell^{\nu)} \partial^2 \right] \int d^4x' iG_2^1(x; x') \tilde{C}_{\text{lin}0}^{0k0\ell}(x') \end{aligned} \quad (128)$$

where  $\tilde{C}_{\text{lin}0}^{\alpha\beta\gamma\delta}(x)$  is the linearized conformally transformed Weyl tensor formed from the 0-th order graviton field  $h_{\mu\nu}^0(x)$ .

## 5.2 Schwinger-Keldysh Formalism

There are various sorts of “effective field equations” corresponding to different definitions of the “effective field”  $h_{\mu\nu}(x)$ . People are most familiar with the in-out effective field equations, which describe the in-out matrix element of the graviton field. That is indeed the best way of describing scattering problems on a flat space background, but it has little relevance for cosmology in which the universe began with a singularity and no one knows how (or even if) it will end. Using the in-out effective field equations for cosmology would have the highly undesirable effect of making evolution at some finite time  $\eta$  depend upon our assumption about the asymptotic future. Further, because the in state will not typically equal the out one, the in-out effective field develops an imaginary part!

The more appropriate cosmological problem is to release the universe in a prepared state at some finite time  $\eta_i$  and then follow the evolution of the expectation value. The Schwinger-Keldysh formalism [23] gives the effective field equations for this situation, and it does so in a way that is almost as simple as the Feynman diagram technology of in-out computations. The salient features are [24]:

- The same Heisenberg field operator  $\varphi(x)$  gives rise to two dummy variables,  $\phi_{\pm}(x)$  in the functional integral formalism. The functional integration over  $\phi_{+}(x)$  implements forward time evolution from the prepared state to some point in the future of the latest observation, while the functional integration over  $\phi_{-}(x)$  implements backwards evolution to the original state.
- Each end of a Schwinger-Keldysh propagator carries a  $\pm$  polarity, corresponding to which of the two dummy fields is meant.
- Vertices and counterterms are either all  $+$  or all  $-$ . The  $+$  vertices and counterterms are identical to those of the in-out formalism, while the  $-$  vertices and counterterms are conjugated.
- Every 1PI N-point function of the in-out formalism gives rise to  $2^N$  Schwinger-Keldysh 1PI N-point functions, corresponding to the two possible polarities for each leg.
- It is the sum of the  $++$  and  $+-$  1PI 2-point functions which appears in linearized effective field equations such as (117).
- For the case of interest to cosmology, in which propagators depend only on the scale factors and the de Sitter length function (5), the four Schwinger-Keldysh propagators follow from the Feynman propagator by simply changing the  $i\varepsilon$  prescription according to the rule:

$$i\Delta_{++}(x; x') \implies y_{++}(x; x) \equiv H^2 aa' \left[ \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\varepsilon)^2 \right], \quad (129)$$

$$i\Delta_{+-}(x; x') \implies y_{+-}(x; x) \equiv H^2 aa' \left[ \|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' + i\varepsilon)^2 \right], \quad (130)$$

$$i\Delta_{-+}(x; x') \implies y_{-+}(x; x) \equiv H^2 aa' \left[ \|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' - i\varepsilon)^2 \right], \quad (131)$$

$$i\Delta_{--}(x; x') \implies y_{--}(x; x) \equiv H^2 aa' \left[ \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| + i\varepsilon)^2 \right]. \quad (132)$$

- Perturbative corrections to using free vacuum as the prepared state correspond to vertices on the initial value surface [24, 25].

It remains only to convert our previous results from the structure functions to Schwinger-Keldysh form. At the one loop order we are working, this is done by taking the in-out result with the replacement  $y(x; x') \rightarrow y_{++}(x; x')$ ,

and then subtracting the in-out result with the replacement  $y(x; x') \rightarrow y_{+-}(x; x')$ . The projectors are not affected at all, so we merely give the three nonzero structure functions of the Schwinger-Keldysh formalism,

$$F_0^1 = \frac{i\kappa^2}{576\pi^3} \left\{ \left[ \frac{\partial^4 - 4H^2 aa' \partial^2}{16} \right] \left[ \left[ \ln\left(\frac{-y}{4aa'}\right) - 1 \right] \Theta \right] - \frac{1}{4} H^2 aa' \ln(aa') \partial^2 \Theta \right. \\ \left. + H^4 a^2 a'^2 \left[ 3 - \frac{1}{4-y} + \frac{3}{4} (2-y) \ln\left(\frac{-y}{4-y}\right) \right] \Theta \right\}, \quad (133)$$

$$F_2^1 = \frac{i\kappa^2}{64\pi^3} \left\{ \left[ \frac{\partial^4 + 20H^2 aa' \partial^2}{240} \right] \left[ \left[ \ln\left(\frac{-y}{4aa'}\right) - 1 \right] \Theta \right] + \frac{H^2 aa' \ln(aa')}{12} \partial^2 \Theta \right. \\ \left. + H^4 a^2 a'^2 \left[ \frac{-\frac{1}{3}}{4-y} - \frac{1}{6} \ln\left(\frac{-y}{4-y}\right) \right] \Theta \right\}, \quad (134)$$

$$G_2^1 = \frac{i\kappa^2}{64\pi^3} \left\{ H^4 a^2 a'^2 \left[ \frac{\frac{4}{3}}{4-y} + \frac{1}{3} \ln\left(\frac{-y}{4-y}\right) \right] \Theta \right\}. \quad (135)$$

In these expressions the symbol  $\Theta$  stands for the  $\theta$ -function which enforces causality,

$$\Theta \equiv \theta\left(\Delta\eta - \|\vec{x} - \vec{x}'\|\right) \quad , \quad \Delta\eta \equiv \eta - \eta' \quad , \quad (136)$$

and we remind the reader that  $-y(x; x')$  is,

$$-y(x; x') = H^2 aa' \left[ \Delta\eta^2 - \|\vec{x} - \vec{x}'\|^2 \right]. \quad (137)$$

Note also that we have set our renormalization scale to  $\mu = \frac{1}{2}H$  in order to facilitate comparison with the results of Refs. [9, 10].

Several points about expressions (133-135) deserve comment. First, the fact that each of the Schwinger-Keldysh structure functions is pure imaginary means that the effective field equation (128) is pure real. This is an important feature of the Schwinger-Keldysh effective field equations which is not shared by the more familiar, in-out effective field equations. A similarly distinctive feature is the causality enforcing  $\theta$ -function (136). One consequence of this causality is that partially integrating spatial derivatives can produce no surface terms provided the interaction begins at some finite time. Partial integration of time derivatives produces no surface terms in the future but it can give rise to surface terms at the initial time. Because perturbative

corrections to the initial state also produce initial time surface integrals it is usual to assume that they cancel the surface terms produced by desired partial time integrations [25], such as those involved in reflecting the primed projectors off of the structure functions and onto the graviton field. However, this has not been checked.

### 5.3 Source for dynamical gravitons

To study dynamical gravitons we set the stress tensor to zero in equation (127). The general solution can be expressed as a superposition of transverse-traceless, spatial plane wave gravitons of the form,

$$h_{\mu\nu}^0(x) = \epsilon_{\mu\nu}(\vec{k})u_0(\eta, k)e^{i\vec{k}\cdot\vec{x}} \quad , \quad u_0(\eta, k) = \frac{H}{\sqrt{2k^3}}\left[1 - \frac{ik}{Ha}\right]e^{-ik\eta} . \quad (138)$$

The polarization tensor  $\epsilon_{\mu\nu}(\vec{k})$  is identical to the one usually employed in flat space. In particular, its temporal components vanish, and it is transverse and traceless,

$$\epsilon_{\mu 0}(\vec{k}) = 0 \quad , \quad k_i \epsilon_{ij}(\vec{k}) = 0 \quad , \quad \epsilon_{ii}(\vec{k}) = 0 . \quad (139)$$

Taken with expressions (122-123), these facts demonstrate the vanishing of the spin zero contributions to one loop effective field equation (128),

$$h_{\mu\nu}^0(x) = \epsilon_{\mu\nu}(\vec{k})u_0(\eta, k)e^{i\vec{k}\cdot\vec{x}} \quad \implies \quad \mathcal{R}^0(x) = 0 = \mathcal{S}^0(x) . \quad (140)$$

For the zeroth order solution (138-139) the only nonzero components of the linearized Weyl tensor are (up to index permutations),

$$\tilde{C}_{\text{lin}0}^{0i0j}(x) = -\frac{1}{4}\epsilon^{ij} \times (\partial_0^2 - k^2)u_0(\eta, k)e^{i\vec{k}\cdot\vec{x}} , \quad (141)$$

$$\tilde{C}_{\text{lin}0}^{0ijk}(x) = -\frac{i}{2}\left(\epsilon^{ij}k^k - \epsilon^{ik}k^j\right) \times \partial_0 u_0(\eta, k)e^{i\vec{k}\cdot\vec{x}} , \quad (142)$$

$$\begin{aligned} \tilde{C}_{\text{lin}0}^{ijkl}(x) &= \frac{1}{2}\left(\epsilon^{ik}k^j k^\ell - \epsilon^{kj}k^\ell k^i + \epsilon^{j\ell}k^i k^k - \epsilon^{\ell i}k^k k^j\right) \times u_0(\eta, k)e^{i\vec{k}\cdot\vec{x}} \\ &\quad - \frac{1}{4}\left(\epsilon^{ik}\delta^{j\ell} - \epsilon^{kj}\delta^{\ell i} + \epsilon^{j\ell}\delta^{ik} - \epsilon^{\ell i}\delta^{kj}\right) \times (\partial_0^2 + k^2)u_0(\eta, k)e^{i\vec{k}\cdot\vec{x}} . \end{aligned} \quad (143)$$

Now make a 3 + 1 decomposition of the first spin two contribution to (128),

$$2\partial_\alpha\partial_\beta\int d^4x' iF_2^1(x; x')\tilde{C}_{\text{lin}0}^{\alpha\mu\beta\nu}(x') = 2\partial_0^2\int d^4x' iF_2^1(x; x')\tilde{C}_{\text{lin}0}^{0\mu 0\nu}(x') \\ -4\partial_0\partial_k\int d^4x' iF_2^1(x; x')\tilde{C}_{\text{lin}0}^{0(\mu\nu)k}(x') + 2\partial_k\partial_\ell\int d^4x' iF_2^1(x; x')\tilde{C}_{\text{lin}0}^{k\mu\ell\nu}(x') , \quad (144)$$

$$= 2\partial_0^2\int d^4x' iF_2^1(x; x')\tilde{C}_{\text{lin}0}^{0\mu 0\nu}(x') \\ -4i\partial_0k_k\int d^4x' iF_2^1(x; x')\tilde{C}_{\text{lin}0}^{0(\mu\nu)k}(x') - 2k_kk_\ell\int d^4x' iF_2^1(x; x')\tilde{C}_{\text{lin}0}^{k\mu\ell\nu}(x') . \quad (145)$$

The analogous expansion for the second spin two contribution is just the last term. From relations (141-143) we see that only the third of the  $\tilde{C}_{\text{lin}0}^{0k0\ell}$  terms survives,

$$\left[\eta^{\mu\nu}\partial_k\partial_\ell - 2\delta_{(k}^{(\mu}\partial^{\nu)}\partial_\ell + \delta_k^{(\mu}\delta_\ell^{\nu)}\partial^2\right]\int d^4x' iG_2^1(x; x')\tilde{C}_{\text{lin}}^{0k0\ell}(x') \\ = -(\partial_0^2 + k^2)\int d^4x' iG_2^1(x; x')\tilde{C}_{\text{lin}}^{0\mu 0\nu}(x') . \quad (146)$$

There is no contribution when either of the indices  $\mu$  or  $\nu$  is temporal. When  $\mu = i$  and  $\nu = j$  equation (128) reads,

$$\partial_\alpha\left[a^2\mathcal{L}^{ij\rho\sigma\alpha\beta}\partial_\beta h_{\rho\sigma}^1(x)\right] = 2\epsilon^{ij}k^2\partial_0\int d^4x' iF_2^1(x; x')\partial'_0 u_0(\eta', k)e^{i\vec{k}\cdot\vec{x}'} \\ + \frac{1}{2}\epsilon^{ij}(\partial_0^2 - k^2)\int d^4x' \left[iF_2^1(x; x') + \frac{i}{2}G_2^1(x; x')\right](\partial_0'^2 - k^2)u_0(\eta', k)e^{i\vec{k}\cdot\vec{x}'} \quad (147)$$

In view of equation (147) we may assume that the one loop correction to the graviton field takes the same form as (138),

$$h_{\mu\nu}^1(x) = \epsilon_{\mu\nu}(\vec{k})u_1(\eta, k)e^{i\vec{k}\cdot\vec{x}} . \quad (148)$$

(This form obviously pertains to all orders.) Substituting (148) into (147), and factoring out the polarization tensor and the trivial spatial dependence, allows us to read off an equation for the one loop correction to mode function,

$$-\frac{1}{2}a^2\left[\partial_0^2 + 2aH\partial_0 + k^2\right]u_1(\eta, k) = 2k^2\partial_0\int d^4x' iF_2^1(x; x')\partial'_0 u_0(\eta', k)e^{-i\vec{k}\cdot\Delta\vec{x}} \\ + \frac{1}{2}(\partial_0^2 - k^2)\int d^4x' \left[iF_2^1(x; x') + \frac{i}{2}G_2^1(x; x')\right](\partial_0'^2 - k^2)u_0(\eta', k)e^{-i\vec{k}\cdot\Delta\vec{x}} , \quad (149)$$

where  $\Delta\vec{x} \equiv \vec{x} - \vec{x}'$ . The 0th order solution (138) implies an important simplification,

$$(\partial_0^2 - k^2)u_0(\eta, k) = -2ik \times \partial_0 u_0(\eta, k) = -2ik \times \frac{H}{\sqrt{2k^3}} \left[ \frac{-k^2}{Ha} \right] e^{-ik\eta}. \quad (150)$$

Relation (150) allows us to combine the various  $F_2^1$  terms in (149),

$$\begin{aligned} -\frac{1}{2}a^2 \left[ \partial_0^2 + 2aH\partial_0 + k^2 \right] u_1(\eta, k) &= k(\partial_0 + ik)^2 \int d^4x' F_2^1(x; x') \partial'_0 u_0(\eta', k) e^{-i\vec{k} \cdot \Delta\vec{x}} \\ &\quad + \frac{1}{2}k(\partial_0^2 - k^2) \int d^4x' G_2^1(x; x') \partial'_0 u_0(\eta', k) e^{-i\vec{k} \cdot \Delta\vec{x}}. \end{aligned} \quad (151)$$

Note that we have not yet made any assumption — beyond homogeneity and isotropy — about the form of the tensor structure functions, so equations (149) and (151) are correct for any contribution to the graviton self-energy, including that of gravitons.

Because we have not corrected the initial state equation [25], (149) and (151) can only be reliably used to infer secular growth, if there is any. Based on the experience of [12], the minimum interesting secular dependence would be  $u_1(\eta, k) \sim \ln(a)/a^2$ , which results in no correction to the tensor power spectrum but does cause the electric components of the one loop linearized Weyl tensor to grow like  $\kappa^2 H^2 \ln(a)$  relative to the classical result (141). To get  $u_1(\eta, k) \sim \ln(a)/a^2$  would require the source terms on the right hand side of (149) and (151) to grow like  $a^2 \ln(a)$ .

The simplest part of the structure functions to analyze is the one implicit in the flat space limit [26],

$$\left( F_2^1(x; x') \right)_{\text{flat}} = \frac{i\kappa^2 \partial^4}{2^{10} \cdot 3 \cdot 5 \cdot \pi^3} \left\{ \theta(\Delta\eta - \Delta x) \left[ \ln \left[ H^2(\Delta\eta^2 - \Delta x^2) \right] - 1 \right] \right\}. \quad (152)$$

Substituting this in (151) and performing the integrations gives,

$$\begin{aligned} k(\partial_0 + ik)^2 \int d^4x' \left( F_2^1(x; x') \right)_{\text{flat}} \partial'_0 u_0(\eta', k) e^{-i\vec{k} \cdot \Delta\vec{x}} \\ = -\frac{\kappa^2(\partial_0^2 + k^2)^2}{2^8 \cdot 3 \cdot 5 \cdot \pi^2} \left\{ \left[ 2 \ln(H\Delta\eta_i) + \int_0^{2k\Delta\eta_i} dt \left[ \frac{e^{it} - 1}{t} \right] \right] u(\eta, k) \right\} \end{aligned} \quad (153)$$

where  $H\Delta\eta_i = 1 - \frac{1}{a}$ . Expression (153) approaches a constant at late times, which would induce irrelevant corrections of the form  $u_1(\eta, k) \sim \frac{1}{a^4}$ . However,

expression (153) can probably be completely cancelled by the same state correction which eliminates its flat space cousin [26].

We might term the remaining parts of the structure functions as “dS” because they contain one or two multiplicative factors of the de Sitter Hubble parameter and scale factors,  $H^2 aa'$ . After a number of tedious manipulations these terms can be expressed as the sum of three double integrals,

$$\begin{aligned}
& k(\partial_0 + ik)^2 \int d^4 x' \left[ F_2^1(x; x') + \frac{1}{2} G_2^1(x; x') \right]_{\text{dS}} \partial'_0 u_0(\eta', k) e^{-i\vec{k} \cdot \Delta \vec{x}} \\
& \quad - ik^2 \partial_0 \int d^4 x' G_2^1(x; x') \partial'_0 u_0(\eta', k) e^{-i\vec{k} \cdot \Delta \vec{x}} \\
& = \frac{i\kappa^2 H k^2 u_0(0, k)}{96\pi^2} (\partial_0 + ik)^2 a \int_{\eta_i}^{\eta} d\eta' e^{-ik\eta'} \left\{ \sin(k\Delta\eta) \ln\left(\frac{\Delta\eta^2}{\eta\eta'}\right) \right. \\
& \quad + \sin\left[k(\eta + \eta')\right] \ln\left(\frac{\eta'}{\eta}\right) + \int_0^1 \frac{dt}{t} \left[ \sin\left[k\Delta\eta(1-2t)\right] - \sin(k\Delta\eta) \right. \\
& \quad \left. \left. - \sin\left[k(\eta + \eta' - 2\eta t)\right] + \sin\left[k(\eta + \eta' - 2\eta' t)\right] \right] \right\} \\
& \quad + \frac{\kappa^2 H k^3 u_0(0, k)}{24\pi^2} \partial_0 a \int_{\eta_i}^{\eta} d\eta' e^{-ik\eta'} \left\{ \left[ \sin\left[k(\eta + \eta')\right] - \frac{\sin(k\eta)}{k\eta} \sin(k\eta') \right] \ln\left(\frac{\eta'}{\eta}\right) \right. \\
& \quad \left. + \int_0^1 \frac{dt}{t} \left[ \sin\left[k(\eta + \eta' - 2\eta' t)\right] - \sin\left[k(\eta + \eta' - 2\eta t)\right] \right] \right\} \\
& \quad + \frac{\kappa^2 H^2 k u_0(0, k)}{48\pi^2} \partial_0 a^2 \int_{\eta_i}^{\eta} d\eta' \frac{e^{-ik\eta'}}{\eta'} \left\{ T(k\Delta\eta) \ln\left(\frac{\Delta\eta^2}{\eta'^2}\right) \right. \\
& \quad + T(k\eta) \cos(k\eta') \ln\left(\frac{\eta'^2}{\eta^2}\right) + \int_0^1 \frac{dt}{t} \left[ T\left[k\Delta\eta(1-2t)\right] - T(k\Delta\eta) \right. \\
& \quad \left. \left. - T\left[k(\eta + \eta' - 2\eta t)\right] + T\left[k(\eta + \eta' - 2\eta' t)\right] \right] \right\}, \quad (154)
\end{aligned}$$

where  $T(x) \equiv \sin(x) - x \cos(x)$ . Further simplifications are possible but the small  $\eta$  behavior of each integrand already shows that none of the three terms can grow faster than  $a^2$  after the derivatives have been acted. Hence we confirm the conclusion of [10] that the inflationary production of massless, minimally coupled scalars makes no significant corrections to dynamical

gravitons at one loop order. Another point to note is that acting the two derivatives on the logarithm of  $\Delta\eta^2$  in the first integral produces terms which diverge at the initial time, which is an indication that they should be absorbed into perturbative corrections to the initial state .

## 6 Discussion

We have developed a noncovariant, but simple, representation for the tensor structure of matter contributions to the self-energy of a conformally re-scaled graviton,  $g_{\mu\nu}^{\text{full}} \equiv a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu})$ , where  $a = -1/H\eta$  is the de Sitter scale factor and  $\kappa^2 \equiv 16\pi G$  is the loop-counting parameter of quantum gravity. Our representation is a sum of four terms, each of which consists of a transverse, 4th order differential operator acting on a structure function,

$$\begin{aligned} -i\left[\mu\nu\Sigma^{\rho\sigma}\right](x;x') &= \mathcal{F}^{\mu\nu}(x) \times \mathcal{F}^{\rho\sigma}(x') \left[F_0(x;x')\right] \\ &+ \mathcal{G}^{\mu\nu}(x) \times \mathcal{G}^{\rho\sigma}(x') \left[G_0(x;x')\right] + \mathcal{F}^{\mu\nu\rho\sigma} \left[F_2(x;x')\right] + \mathcal{G}^{\mu\nu\rho\sigma} \left[G_2(x;x')\right] . \end{aligned} \quad (155)$$

There are two scalar contributions and two tensor ones. The scalar operator  $\mathcal{F}^{\mu\nu}$  is given in expression (33), while  $\mathcal{G}^{\mu\nu}$  is given in (36). The transverse and traceless operators  $\mathcal{F}^{\mu\nu\rho\sigma}$  and  $\mathcal{G}^{\mu\nu\rho\sigma}$  — see expressions (45) and (46), respectively — are constructed by contracting different 8-index tensors into the product of primed and unprimed Weyl operators  $\mathcal{C}_{\alpha\beta\gamma\delta}{}^{\mu\nu}$ , for which see (40). The two  $F$ -type structure functions,  $F_0(x;x')$  and  $F_2(x;x')$  survive in the flat space limit, whereas  $G_0(x;x')$  and  $G_2(x;x')$  vanish in that limit.

In section 4.1 we have also given a general procedure for determining the four structure functions from the primitive form of the self-energy. One first finds the scalar structure functions  $F_0(x;x')$  and  $G_0(x;x')$  by taking the trace on one index group — say  $\rho$  and  $\sigma$  — and then examining the  $\mu\nu = 0i$  and  $\mu\nu = jk$  components. The resulting expressions for  $F_0(x;x')$  and  $G_0(x;x')$  are (56) and (57), respectively. Once the scalar structure functions are known one finds the tensor structure functions by examining the case for which all indices differ. The resulting expressions for  $F_2(x;x')$  and  $G_2(x;x')$  are (66) and (67).

As an example, we constructed the structure functions for the one loop contribution from a massless, minimally coupled scalar [9]. For this case the structure function  $G_0(x;x')$  happens to vanish. Our results for the other structure functions are expression (99) for  $F_0(x;x')$ , expression (116) for



$F_2(x; x')$ , and expression (114) for  $G_2(x; x')$ . Because this model shows no physical breaking of de Sitter invariance one could have employed a de Sitter covariant representation of the self-energy, however, our noncovariant representation is much simpler. Any doubts about this can be quickly settled by comparison with the *full page* expressions (234) and (258) for the de Sitter invariant structure functions [9].

Our noncovariant representation is also much easier to use in the effective field equations than the covariant one. Expression (128) gives a formula for the one loop graviton field. When specialized to the case of dynamical gravitons it becomes (151). One nice property of this representation is that surface terms really fall off like powers of the inflationary scale factor, whereas this is not always true in de Sitter invariant formulations [19]. We used this property to check that massless, minimally coupled scalars really have no significant effect on dynamical gravitons at one loop. The previous de Sitter invariant analysis was based on the assumption that a certain surface term can be ignored, either because it falls off or because it can be absorbed into corrections of the initial state [10]. Our new result confirms this assumption.

Although the point of our paper was to develop a new representation for the graviton self-energy, it is worth commenting on the physics we found in applying it. We used the new formalism to demonstrate the absence of any effect on dynamical gravitons from massless, minimally coupled (MMC) scalars. This is not because there are no scalars. Inflation produces a veritable sea of infrared scalars which mediate significant effects on themselves [2], on photons [3, 4], and on fermions [5]. The difference between those cases and the null result we found for gravitons is the presence of non-derivative interactions. The inflationary production of MMC scalars engenders a steady increase in the magnitude of the scalar field strength, so particles which couple to undifferentiated scalars experience an effect. In contrast, gravitons couple minimally only to *differentiated* scalars, and there is no build-up of that.

Although we have not done so, one can also employ the graviton self-energy to study the force of gravity. In this case there must be some effect from MMC scalars because even the flat space limit shows a correction to the Newtonian potential at one loop [27],

$$\Phi(r) = \frac{GM}{r} \left\{ 1 + \frac{\hbar G}{20\pi c^3 r^2} + \dots \right\}. \quad (156)$$

The physical origin of this effect is that the classical potential of a point mass

tends to attract virtual scalars, which adds to the source. During inflation there should be an additional, secular effect as newly created scalars accrete onto the source. It is conceivable that this secular effect shows up as a one loop correction of the form,

$$\Delta\Phi(t, r) = \frac{GM}{r} \times (\text{Constant}) \times \frac{\hbar GH^2}{c^5} \times \ln(a) . \quad (157)$$

At this order such a correction would be indistinguishable from a secular increase of the Newton constant, which would be a fascinating result. We now have a formalism of sufficient power and simplicity to confirm or refute this possibility. We can also identify what part of the structure functions is most likely to cause it: the terms  $6\ln(\frac{y}{4})$  and  $-\frac{3}{2}y\Psi(y)$  on the second line of expression (99) for the scalar structure function  $F_0(x; x')$ . Recall that  $F_0(x; x')$  drops out of the effective field equations for dynamical gravitons, but it contributes to the equations for the force of gravity. Had either of the tensor structure functions (114) and (116) possessed the same sort of terms we would have found secular enhancements in the curvature carried by dynamical gravitons.

Finally, two extensions of our formalism are worth noting. First, this same representation (155) can be used as well for graviton contributions in transverse gauge. In other gauges — for example, [28] — the self-energy is not transverse and one would need to revise the scalar terms but the form of the spin two contributions would not be changed. Second, the same representation (155) should apply to any homogeneous, isotropic and spatially flat geometry, with only some slight generalizations to the two scalar terms.

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## 7 Appendix: The Projectors $\mathcal{F}^{\mu\nu\rho\sigma}$ and $\mathcal{G}^{\mu\nu\rho\sigma}$

Substituting expressions (40-43) into (45) and performing some tedious algebra gives a relatively simple form for  $\mathcal{F}^{\mu\nu\rho\sigma}$  that is manifestly transverse on each index group,

$$\mathcal{F}^{\mu\nu\rho\sigma} = \frac{1}{2} \left( \mathcal{P}^{\mu\rho} \mathcal{P}^{\nu\sigma} + \mathcal{P}^{\mu\sigma} \mathcal{P}^{\nu\rho} \right) - \frac{4}{D-2} \mathcal{D}_{\alpha\beta}{}^{\mu\nu} \mathcal{D}'^{\alpha\beta\rho\sigma} + \frac{2\mathcal{D}^{\mu\nu} \mathcal{D}'^{\rho\sigma}}{(D-1)(D-2)}. \quad (158)$$

Here the projector  $\mathcal{P}^{\mu\rho}$  is the same one that acts on the structure function  $F(x; x')$  in the vacuum polarization,

$$\mathcal{P}^{\mu\rho} \equiv \eta^{\mu\rho} \partial' \cdot \partial - \partial'^\mu \partial^\rho. \quad (159)$$

Expanding out the first term on the right hand side of (158) gives,

$$\frac{1}{2} \left( \mathcal{P}^{\mu\rho} \mathcal{P}^{\nu\sigma} + \mathcal{P}^{\mu\sigma} \mathcal{P}^{\nu\rho} \right) = \eta^{\mu(\rho} \eta^{\sigma)\nu} (\partial \cdot \partial')^2 - 2\partial'^{(\mu} \eta^{\nu)(\rho} \partial^\sigma \partial \cdot \partial' + \partial'^\mu \partial^\nu \partial^\rho \partial^\sigma. \quad (160)$$

A similarly explicit form for the contraction in the middle term of (158) is,

$$\begin{aligned} 4\mathcal{D}_{\alpha\beta}{}^{\mu\nu} \mathcal{D}'^{\alpha\beta\rho\sigma} &= \eta^{\mu\nu} \eta^{\rho\sigma} (\partial \cdot \partial')^2 + \eta^{\rho(\mu} \eta^{\nu)\sigma} \partial^2 \partial'^2 + \eta^{\mu\nu} \left[ \partial^\rho \partial^\sigma \partial'^2 - 2\partial^{(\rho} \partial'^{\sigma)} \partial \cdot \partial' \right] \\ &+ \eta^{\rho\sigma} \left[ \partial'^\mu \partial'^\nu \partial^2 - 2\partial^{(\mu} \partial'^{\nu)} \partial \cdot \partial' \right] - 2\partial^{(\mu} \eta^{\nu)(\rho} \partial^\sigma \partial'^2 - 2\partial'^{(\mu} \eta^{\nu)(\rho} \partial^\sigma \partial^2 \\ &+ 2\partial^{(\mu} \eta^{\nu)(\rho} \partial'^\sigma \partial \cdot \partial' + 2\partial'^{(\mu} \partial'^{\nu)} \partial^{(\rho} \partial'^{\sigma)} \partial. \end{aligned} \quad (161)$$

The “essentially spatial” projector  $\mathcal{G}^{\mu\nu\rho\sigma}$  requires contractions of the linearized Riemann operator (41) into one or two spatial metrics  $\bar{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta_0^\mu \delta_0^\nu$ ,

$$\mathcal{D}_{1\beta\delta}{}^{\mu\nu} \equiv \eta^{\alpha\gamma} \mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu} = -\frac{1}{2} \left[ \eta^{\mu\nu} \bar{\partial}_\beta \bar{\partial}_\delta - 2\partial^{(\mu} \bar{\partial}^{\nu)}_{(\beta} \bar{\partial}_{\delta)} + \bar{\partial}^{(\mu} \bar{\partial}^{\nu)}_{\delta} \partial^2 \right], \quad (162)$$

$$\mathcal{D}_1{}^{\mu\nu} \equiv \eta^{\alpha\gamma} \bar{\eta}^{\beta\delta} \mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu} = -\frac{1}{2} \left[ \eta^{\mu\nu} \nabla^2 - 2\partial^{(\mu} \bar{\partial}^{\nu)} + \bar{\eta}^{\mu\nu} \partial^2 \right], \quad (163)$$

$$\mathcal{D}_2{}^{\mu\nu} \equiv \bar{\eta}^{\alpha\gamma} \bar{\eta}^{\beta\delta} \mathcal{D}_{\alpha\beta\gamma\delta}{}^{\mu\nu} = \bar{\partial}^\mu \bar{\partial}^\nu - \bar{\eta}^{\mu\nu} \nabla^2. \quad (164)$$

Substituting (40-43) into (46) and working through the tensor algebra gives,

$$\begin{aligned} \mathcal{G}^{\mu\nu\rho\sigma} &= \frac{1}{2} \left[ \mathcal{P}_1{}^{\mu\rho} \mathcal{P}_1{}^{\nu\sigma} + \mathcal{P}_1{}^{\mu\sigma} \mathcal{P}_1{}^{\nu\rho} \right] + \frac{4(D-3)}{(D-2)^2} \mathcal{D}_{\alpha\beta}{}^{\mu\nu} \mathcal{D}'^{\alpha\beta\rho\sigma} \\ &- \frac{4}{D-2} \left[ \mathcal{D}_{1\alpha\beta}{}^{\mu\nu} \mathcal{D}'^{\alpha\beta\rho\sigma} + \mathcal{D}_{\alpha\beta}{}^{\mu\nu} \mathcal{D}'_1{}^{\alpha\beta\rho\sigma} \right] + \frac{4}{(D-2)^2} \mathcal{D}_1{}^{\mu\nu} \mathcal{D}'_1{}^{\rho\sigma} \\ &+ \frac{[2\mathcal{D}^{\mu\nu} \mathcal{D}'^{\rho\sigma} - 4\mathcal{D}_1{}^{\mu\nu} \mathcal{D}'^{\rho\sigma} - 4\mathcal{D}^{\mu\nu} \mathcal{D}'_1{}^{\rho\sigma} + 2\mathcal{D}^{\mu\nu} \mathcal{D}'_2{}^{\rho\sigma} + 2\mathcal{D}_2{}^{\mu\nu} \mathcal{D}'^{\rho\sigma}]}{(D-1)(D-2)}, \end{aligned} \quad (165)$$

where  $\mathcal{P}_1^{\mu\rho}$  is the same spatial transverse projector that appears on the structure function  $G(x; x')$  in the vacuum polarization,

$$\mathcal{P}_1^{\mu\rho} \equiv \bar{\eta}^{\mu\rho} \nabla' \cdot \nabla - \bar{\partial}'^\mu \bar{\partial}^\rho. \quad (166)$$

The explicit form of the first term on the right of (165) is,

$$\frac{1}{2} \left( \mathcal{P}_1^{\mu\rho} \mathcal{P}'_1{}^{\nu\sigma} + \mathcal{P}_1^{\mu\sigma} \mathcal{P}'_1{}^{\nu\rho} \right) = \bar{\eta}^{\mu(\rho} \bar{\eta}^{\sigma)\nu} (\nabla \cdot \nabla')^2 - 2 \bar{\partial}'^{(\mu} \bar{\eta}^{\nu)(\rho} \bar{\partial}^{\sigma)} \nabla \cdot \nabla' + \bar{\partial}'^\mu \bar{\partial}'^\nu \bar{\partial}^\rho \bar{\partial}^\sigma. \quad (167)$$

The explicit forms for the three contractions in (165) are,

$$\begin{aligned} 4\mathcal{D}_{\alpha\beta}^{\mu\nu} \mathcal{D}'^{\alpha\beta\rho\sigma} &= \eta^{\mu\nu} \eta^{\rho\sigma} (\nabla \cdot \nabla')^2 + \eta^{\mu\nu} [\bar{\partial}^\rho \bar{\partial}^\sigma \partial'^2 - 2\bar{\partial}^{(\rho} \partial'^{\sigma)} \nabla \cdot \nabla'] \\ &\quad + \bar{\eta}^{\mu(\rho} \bar{\eta}^{\sigma)\nu} \partial^2 \partial'^2 + \eta^{\rho\sigma} [\bar{\partial}'^\mu \bar{\partial}'^\nu \partial^2 - 2\partial^{(\mu} \bar{\partial}'^{\nu)} \nabla \cdot \nabla'] - 2\bar{\partial}'^{(\mu} \bar{\eta}^{\nu)(\rho} \partial'^{\sigma)} \partial^2 \\ &\quad - 2\partial^{(\mu} \bar{\eta}^{\nu)(\rho} \bar{\partial}^{\sigma)} \partial'^2 + 2\partial^{(\mu} \bar{\eta}^{\nu)(\rho} \partial'^{\sigma)} \nabla \cdot \nabla' + 2\partial^{(\mu} \bar{\partial}'^{\nu)} \bar{\partial}^{(\rho} \partial'^{\sigma)} \partial^2, \end{aligned} \quad (168)$$

$$\begin{aligned} 4\mathcal{D}_{1\alpha\beta}^{\mu\nu} \mathcal{D}'^{\alpha\beta\rho\sigma} &= \bar{\eta}^{\mu\nu} \eta^{\rho\sigma} (\nabla \cdot \nabla')^2 + \bar{\eta}^{\mu\nu} [\bar{\partial}^\rho \bar{\partial}^\sigma \partial'^2 - 2\bar{\partial}^{(\rho} \partial'^{\sigma)} \nabla \cdot \nabla'] \\ &\quad + \bar{\eta}^{\mu(\rho} \bar{\eta}^{\sigma)\nu} \nabla^2 \partial'^2 + \eta^{\rho\sigma} [\bar{\partial}'^\mu \bar{\partial}'^\nu \nabla^2 - 2\bar{\partial}^{(\mu} \bar{\partial}'^{\nu)} \nabla \cdot \nabla'] - 2\bar{\partial}^{(\mu} \bar{\eta}^{\nu)(\rho} \bar{\partial}^{\sigma)} \partial'^2 \\ &\quad - 2\bar{\partial}'^{(\mu} \bar{\eta}^{\nu)(\rho} \partial'^{\sigma)} \nabla^2 + 2\bar{\partial}^{(\mu} \bar{\eta}^{\nu)(\rho} \partial'^{\sigma)} \nabla \cdot \nabla' + 2\bar{\partial}^{(\mu} \bar{\partial}'^{\nu)} \bar{\partial}^{(\rho} \partial'^{\sigma)} \partial'^2, \end{aligned} \quad (169)$$

$$\begin{aligned} 4\mathcal{D}_{\alpha\beta}^{\mu\nu} \mathcal{D}'_1{}^{\alpha\beta\rho\sigma} &= \eta^{\mu\nu} \bar{\eta}^{\rho\sigma} (\nabla \cdot \nabla')^2 + \eta^{\mu\nu} [\bar{\partial}^\rho \bar{\partial}^\sigma \nabla'^2 - 2\bar{\partial}^{(\rho} \bar{\partial}^{\sigma)} \nabla \cdot \nabla'] \\ &\quad + \bar{\eta}^{\mu(\rho} \bar{\eta}^{\sigma)\nu} \partial^2 \nabla'^2 + \bar{\eta}^{\rho\sigma} [\bar{\partial}'^\mu \bar{\partial}'^\nu \partial^2 - 2\partial^{(\mu} \bar{\partial}'^{\nu)} \nabla \cdot \nabla'] - 2\partial^{(\mu} \bar{\eta}^{\nu)(\rho} \bar{\partial}^{\sigma)} \partial^2 \\ &\quad - 2\partial^{(\mu} \bar{\eta}^{\nu)(\rho} \bar{\partial}^{\sigma)} \nabla'^2 + 2\partial^{(\mu} \bar{\eta}^{\nu)(\rho} \bar{\partial}^{\sigma)} \nabla \cdot \nabla' + 2\partial^{(\mu} \bar{\partial}'^{\nu)} \bar{\partial}^{(\rho} \bar{\partial}^{\sigma)} \partial'^2. \end{aligned} \quad (170)$$

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